Table of Content

2. Operations Planning

2.1 Demand Forecasting

Questions For Classroom Discussion

Question 1

Following are the details of amount spend on advertising and estimated sales.

Year	Advertisement	Sales
2012	25	254
2013	32	306
2014	44	410
2015	52	452
2016	60	528
2017	75	664

Question 2

- ◈ Write down, the regression equation y on x ?
- ◈ Write down, the regression equation x on y ?
- ◈ What is the estimated sales when amount spend on advertising is 90 ?
- ◈ What is the amount to be spend to generate a sale of 1000?

Question 3 **Previous Year Question**

It is observed that there exists a relationship between Expenditure on Advertising and the Annual Sales. The details for last six years are as follows :

Year	Advertisement	Sales
2004	1	18
2005	2	23
2006	4	32
2007	3	28
2008	10	38
2009	4	29

Estimate the annual Sales when Expenditure on Advertising is 5 crores?

Question 4 **Previous Year Question**

The following is the demand for Product A in 5 towns :

Population (In Lakhs)	Demand (1000)
9	12
5	20
8	15
5	10
3	5

Estimate the demand for Product A for a town with a population of 10 Lakhs?

Time Series Model

Question 5 **Previous Year Question**

From the following time series data of sales of Refrigerators, project the sales for the year 2010

Year	Sales
2002	90
2003	100
2004	102
2005	93
2006	104
2007	109
2008	102

Question 6 **Previous Year Question**

With the help of the following data, project the trend of sales for the next 5 years

Year	Sales
2002	120
2003	130
2004	135
2005	140
2006	150
2007	165

Direct Method

$$a=\frac{\sum y}{n}$$

$$b=\frac{\sum xy}{\sum x^2}$$

If the number of years is ODD (5 Year, 7 year, 9 year)

Question 7 **Previous Year Question**

For Question No. 6, we are using the direct method. This is the same question we have previously solved as Question No. 4.

From the following time series data of sales of Refrigerators, project the sales for the year 2010

Year	Sales
2002	90
2003	1000
2004	102
2005	93
2006	104
2007	109
2008	102

If the number of years is EVEN (4Year, 6year, 8 Year….

With the help of the following data, project the trend of sales for the next 5 years

Year	Sales
2002	120
2003	130
2004	135
2005	140
2006	150
2007	165

Illustration From Study Material

Illustration 1:

From the following time series data of sale project the sales for the next three years.

Year	2017	2018	2019	2020	2021	2022	2023
Sales (Rs.000 units)	80	90	92	83	94	99	92

Answer

Computation of Trend Values

Years	**Time Deviation from 2020 X**	**Sales in (Rs. 000 units) Y**	**Squares of time dev. X^2**	**Product of time deviations and sales XY**
2017	-3	80	9	-240
2018	-2	90	4	-180
2019	-1	92	1	-92
2020	0	83	0	0
2021	+1	94	1	+94
2022	+2	99	4	+198
2023	+3	92	9	+276
n = 7	$\Sigma X = 0$	$\Sigma Y = 630$	$\Sigma X^2 = 28$	$\Sigma XY = + 56[$

Regression equation of Y on X

$Y = a + bX$

To find the values of a and b

$$a = \frac{\sum Y}{n} = \frac{630}{7} = 09$$

$$b = \frac{\sum XY}{\sum X^2} = \frac{56}{28} = 2$$

Hence regression equation comes to $Y = 90 + 2X$. With the help of this equation we can project the trend values for the next three years, i.e. 2024, 2025 and 2026.

$Y_{2024} = 90 + 2(4) = 90 + 8 = 98$ (000) units.

$Y_{2025} = 90 + 2(5) = 90 + 10 = 100$ (000) units.

$Y_{2026} = 90 + 2(6) = 90 + 12 = 102$ (000) units.

Illustration 2:

With the help of following data project the trend of sales for the next five years:

Years	2016	2017	2018	2019	2020	2021
Sales (in lakhs)	100	110	115	120	135	140

Answer

Computation of trend values of sales

Year	**Time deviations from the middle of 2004 and 2005 assuming 6 months = 1 unit**	**Sales (in lakh Rs.)**	**Squares of time deviation**	**Product of time deviation and sales**
	X	Y	X2	XY
2016	-5	100	25	-500
2017	-3	110	9	-330
2018	-1	115	1	-115
2019	+1	120	1	+120
2020	+3	135	9	+405
2021	+ 5	140	25	+700

n = 6	ΣX = 0	ΣY = 720	$\Sigma X^2 = 70$	ΣXY = 280

Regression equation of Y on X:

Y = a + bX

To find the values of a and b

$$a = \frac{\sum Y}{n} = \frac{720}{6} = 120$$

$$b = \frac{\sum XY}{\sum X^2} = \frac{280}{70} = 4$$

Hence regression equation comes to Y = 120 + 4X

Sales forecast for the next years, i.e., 2022-26

Y_{2022} = 120 + 4 (+7) = 120 + 28 = Rs. 148 lakhs

Y_{2023} = 120 + 4 (+9) = 120 + 36 = Rs. 156 lakhs

Y_{2024} = 120 + 4 (+11) = 120 + 44 = Rs. 164 lakhs.

Y_{2025} = 120 + 4 (+13) = 120 + 52 = Rs. 172 lakhs.

Y_{2026} = 120 + 4 (+15) = 120 + 60 = Rs. 180 lakhs.

Illustration 3:

An investigation into the demand for colour TV sets in 5 towns has resulted in the following data:

Population of the town (in lakhs)	X:	5	7	8	11	14
No of TV sets demanded (in thousands)	Y:	9	13	11	15	19

Fit a linear regression of Y on X and estimate the demand for CTV sets for two towns with a population of 10 lakhs and 20 lakhs.

Answer

Computation of trend values

Population (in lakhs)	**Sales of CTV (in thousands)**	**Squares of the population**	**Product of population and sales of colour TV**
X	Y	X^2	XY
5	9	25	45
7	13	49	91
8	11	64	88
11	15	121	165
14	19	196	266
ΣX = 45	Σy = 67	$\Sigma X^2 = 455$	ΣXY = 655

Regression equation of Y on X

Y = a + bX

To find the values of a and b, the following two equations are to be solved

ΣY = na + bΣX (i)

ΣXY = aΣX + bΣX2 (ii)

By putting the values we get

67 = 5a + 45b (iii)

655 = 45a + 455b (iv)

Multiplying equation (iii) by 9 and putting it as no. (v) we get,

603 = 45a + 405b ... (v)

By deducting equation (v) from equation (iv); we get 52 = 50b

$$b = \frac{52}{60} = 1.04$$

By putting the value of b in equation (iii), we get

67 = 5a + 45 × 1.04

or, 67 = 5a + 46.80

or, 67-46.80 = 5a

or, 5a = 20.20

or, $a = \frac{20.20}{5}$

or; a = 4.04

Now by putting the values of a and b the required regression equation of Y on X, is

$$Y = a + bX \text{ or, } Y = 4.04 + 1.04X$$

When X = 10 lakhs than

$$Y = 4.04 + 1.04 (10)$$

or, Y = 4.04 + 10.40 or 14.44 thousand CTV sets.

Similarly for town having population of 20 lakhs, by putting the value of X = 20 lakhs in regression equation

$$Y = 4.04 + 1.04 (20)$$

= 4.04 + 20.80 = 24.84 thousands CTV sets.

Hence expected demand for CTV for two towns will be 14.44 thousand and 24.84 thousand CTV sets.

Illustration 4:

An investigation into the use of scooters in 5 towns has resulted in the following data: Population in town

Population in town (in lakhs)	(X)	4	6	7	10	13
No. of scooters	(Y)	4,400	6,600	5,700	8,000	10,300

Fit a linear regression of Y on X and estimate the number of scooters to be found in a town with a population of 16 lakhs.

Answer:

Computation of trend value

Population (in lakhs) X	**No. of scooters demanded Y**	**Squares of population X^2**	**Product of population and No. of scooters demanded XY**
4	4,400	16	17,600
6	6,600	36	39,600
7	5,700	49	39,900
10	8,000	100	80,000
13	10,300	169	1,33,900
$\Sigma X = 40$	$\Sigma Y = 35,000$	$\Sigma X^2 = 370$	$\Sigma XY = 3,11,000$

Regression equation of Y on X

Y = a + bX

To find the values of a and b we will have to solve the following two equations

$\Sigma Y = na + b\Sigma X$ ... (i)

$\Sigma XY = a\Sigma X + b\Sigma X2$ .... (ii)

By putting the values, we get

35,000 = 5a + 40b ... (iii)

3,11,000 = 40a + 370b ... (iv)

By multiplying equation no. (iii) by 8 putting as equation (v) we get,

2,80,000 = 40a + 320b ... (v)

By subtracting equation (v) from equation (iv), we get

31,000 = 50b

or, 50b = 31,000

or, $b = \frac{310}{50} = 620$

By substituting the value of b in equation no. (iii), we get

$$35{,}000 = 5a + 40b$$

Or $35{,}000 = 5a + 40 \times 620$

or $35{,}000 = 5a + 24{,}800$

or $10{,}200 = 5a$

or $a = \frac{10200}{5} = 2040$

Now putting the values of a and b the required regression equation of Y on X, is

$$Y = a + bX \text{ or, } Y = 2040 + 620\,X$$

When X = 16 lakhs then $Y = 2040 + 620\,(16)$

or $Y = 2040 + 9920$

or $Y = 11{,}960$

Hence, the expected demand of scooters for a town with a population of 16 lakhs will be 11,960 scooters.

2.2 Capacity Planning

Questions For Classroom Discussion

Question 1

A worker works for 8 hours in each shift, but during that period, he had clocked for 7 hours on the job. Calculate his utilization.

Question 2

Standard time for a task is 8 hours. Calculate the efficiency of a workman in the following cases.
A) Worker Completes the job in 10 Hours
B) Worker completes the job in 6 Hours
C) Worker completes the job in 8 Hours.

Question 3

A worker is employed for 12 hours. During this period, he takes 8 hours to complete a job with a standard time of 7 hours. Calculate the productivity of the worker as a percentage.

Question 4

A department works on 8 hours shift, 250 days a year and has the usage data of a machine as given below.

Product	Annual Demand	Processing Time
X	300	4
Y	400	6
Z	500	3

Q1: What is the Annual Production capacity of one machine in Hours ?
Q2: What is the Total Processing time required to produce products X,Y,Z is
Q3: Determine Number of Machines Required?

Question 5 **Home Work**

A department works on 8 hours shift, 250 days a year and has the usage data of a machine, as given below:

Product	Annual Demand	Processing Time
A	600	4
B	800	6
C	1000	3

Compute
Q1) Annual Production capacity of a single machine
Q2) What is the total annual processing time required to produce products A,B,C
Q3) Determine the number of machines required ?

Question 6

A steel Plant has a design capacity of 50,000 tons of steels per day, effective capacity of 40,000 tons of steels per day and an actual output of 36,000 tons of steels per day. Compute the efficiency of the plant and its utilization.

Question 7

An item is produced in a plant having a fixed cost of Rs 6000 per month, Variable cost of Rs 2 Per unit and Selling price of Rs 7 per unit. Determine
Q1) The break even Volume
Q2) If 1000 units are produced and sold in a month, what would be the profit?

Q3) How many units should be produced to earn a profit of Rs 4000 per month.

Question 8

A manager has to decide about the number of machines to be purchased. He has three options, i.e. Purchasing one, or two or three machines. The data are given below.

Number of Machine	Annual Fixed Cost	Corresponding range of Output
One	12,000	0 to 300
Two	15,000	301 to 600
Three	21,000	601 to 900

Variable cost is Rs 20 per unit and Revenue is Rs.50 per unit.

a) Determine the Break Even point for each range.
b) If Projected demand is between 600 to 650 units, How many machines should the manager purchase?

Question 9

Previous Year Question

A manager has to decide about the number of machines to be purchased. He has three options, i.e. Purchasing one, or two or three machines. The data are given below.

Number of Machine	Annual Fixed cost	Corresponding range of Output
One	10,000	0 to 400
Two	12,000	401 to 700
Three	20,000	701 to 1000

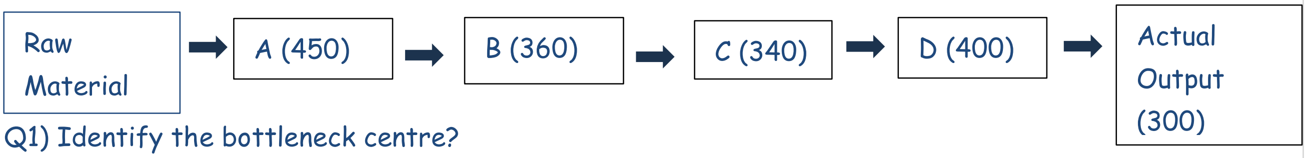
Variable cost is Rs 20 per unit and Revenue is Rs.40 per unit.

a) Determine the Break Even point for each range.
b) If Projected demand is between 700 to 750 units, How many machines should the manager purchase?

June 2010 Q3(a) , 3+2 Marks

Question 10

A firm has four work centres A,B,C, &D in series with individual capacities in units per day shown in the figure below.



Q1) Identify the bottleneck centre?
Q2) What is the system Capacity?
Q3) What is the system Efficiency?

A firm has 5 work centres A,B,C,D,E in series and Actual output of 500 units. Compute
1) System Capacity
2) Efficiency of production line

Work Station	A	B	C	D	E
Capacity/ Shift	600	650	650	550	600

Question 11

A company is planning to start an assembly unit of television sets must decide on the location of its plant at any of the three cities, viz, Kolkata, Delhi or Mumbai. The extent of fixed and variable costs for each of these locations are estimated to be as under

Locations	Kolkata	Delhi	Mumbai
Fixed cost	3,000,000	5,000,000	2,500,000
Variable Cost	300	200	350

Expected Selling Price Rs 700

Q1) Break even point for each locations?

Q2) The range of annual production/ Sales volume for each location is most suitable.

Q3) Which one of the three location is best suitable for a production or sales volume of 18000 units?

Question 12 **Previous Year Question**

The fixed cost for the production of particular item is Rs. 200 per month. Its variable cost being Rs. 3 per unit and its sale price being Rs. 7 per unit,

Q1) Determine its break-even volume.

Q2) What would be the profit if 2,000 such units were sold in a month?

Q3) How many such units should be sold to earn a profit of Rs. 3,000 per month?

Answer

Q1) 50 Units

Q2) Rs 7800

Q3) 800 Units

Question 13 **Previous Year Question**

A company is planning to undertake the production of medical testing equipment and has to decide on the location of the plant. Two locations are being considered, namely, A and B. The fixed costs of two locations are estimated to be Rs. 25 lakhs and Rs. 30 lakhs respectively. The variable costs are Rs. 300 and Rs. 250 per unit respectively. The average sale price of the equipment is Rs. 550 per unit.

Find the range of annual production/sales volume for which each location is most suitable.

Answers

Break Even Point for

A is 10,000 and B 10,000

Since Contribution per Units from Location B is more than A, For above 10,000 units, Location B is preferred.

Illustration From Study Material

Illustration 5:

A department works on 8 hours shift, 250 days a year and has the usage data of a machine, as given below:

Product	Annual demand (units)	Processing time (standard time in hours)
X	300	4.0
Y	400	6.0
Z	500	3.0

Determine the number of machines required.

Answer

Step 1: Calculate the processing time needed in hours to produce product x, y and z in the quantities demanded using the standard time data.

Product	**Annual demand (units)**	**Standard processing time per unit (Hrs.)**	**Processing time needed (Hrs.)**
X	300	4.0	300 x 4 = 1200 Hrs.
Y	400	6.0	400 x 6 = 2400 Hrs.
Z	500	3.0	500 × 3 = 1500 Hrs.
			Total = 5100 Hrs

Step 2 : Annual production capacity of one machine in standard hours = 8 × 250 = 2000 hours per year

Step 3 : Number of machines required

$$= \frac{\text{Work load per year}}{\text{Production capacity per machine}} = \frac{5100}{2000} = 2.55 = 3 \text{ machines}$$

Illustration 6:

A steel plant has a design capacity of 50,000 tons of steel per day, effective capacity of 40,000 tons of steel per day and an actual output of 36,000 tons of steel per day. Compute the efficiency of the plant and its utilisation.

Answer:

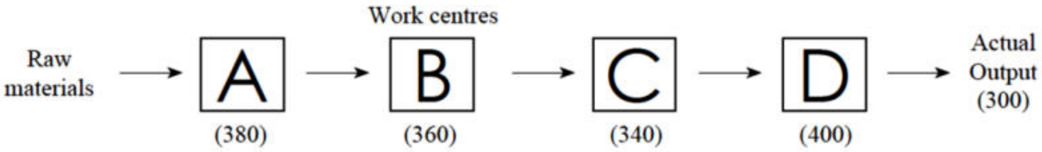
Actual output

$$\text{Efficiency of the plant} = \frac{\text{Actual output}}{\text{Effective Capacity}} = \left(\frac{36000}{40000}\right) \times 100 = 90\%$$

$$\text{Utilisation} = \left(\frac{\text{Actual output}}{\text{Design Capacity}}\right) = \left(\frac{36000}{50000}\right) \times 100 = 72\%$$

Illustration 7:

A firm has four work centres, A, B-n, C & D, in series with individual capacities in units per day shown in the figure below.



(i) Identify the bottle neck centre.
(ii) What is the system capacity?
(iii) What is the system efficiency?

Answer

(i) The bottle neck centre is the work centre having the minimum capacity. Hence, work centre 'C' is the bottleneck centre.

(ii) System capacity is the maximum units that are possible to produce in the system as a whole. Hence, system capacity is the capacity of the bottle neck centre i.e., 340 units.

(iii) System efficiency = $\frac{\text{Actual output}}{\text{System Capacity}} = \frac{300}{400} \times 100$ (i.e.,maximum posiible output) = 88.23%

Illustration 8:

A manager has to decide about the number of machines to be purchased. He has three options i.e., purchasing one, or two or three machines. The data are given below.

Number of Machine	Annual fixed cost	Corresponding range of output
One	Rs. 12,000	0 to 300
Two	Rs. 15,000	301 to 600
Three	Rs. 21,000	601 to 900

Variable cost is Rs. 20 per unit and revenue is Rs. 50 per unit

(a) Determine the break-even point for each range

(b) If projected demand is between 600 and 650 units how many machines should the manager purchase?

Answer

(i) Break-even point

Let Q be the break even point.

FC = Fixed cost, R = Revenue per unit, VC = Variable cost

At, BEP, TR = FC + TVC

or, Revenue p.u × Q = FC + VCp.u. × Q

Q (R - VC) = FC

$$Q = \frac{FC}{R-VC}$$

Let Q1 be the break-even-point for one machine option

Then, $Q1 = \frac{1200}{(50-20)} = \frac{1200}{30} = 400$units

(Not within the range of 0 to 300)

Let Q2 be the break-even-point for two machines option.

Then $Q2 = \frac{1500}{(50-20)} = \frac{1500}{30} = 500$ units

(within the range of 301 to 600)

Let Q3 be the break-even-point for three machines option.

Then $Q1 = \frac{21000}{(50-20)} = \frac{21000}{30} = 700$ units

(with in the range of 601 to 900)

(ii) The projected demand is between 600 to 650 units.

The break even point for single machine option (i.e., 400 units) is not feasible because it exceeds the range of volume that can be produced with one machine (i.e., 0 to 300).

Also, the break even point for 3 machines is 700 units which is more than the upper limit of projected demand of 600 to 650 units and hence not feasible. For 2 machines option the break even volume is 500 units and volume range is 301 to 600.

Hence, the demand of 600 can be met with 2 machines and profit is earned because the production volume of 600 is more than the break even volume of 500. If the manager wants to produce 650 units with 3 machines, there will be loss because the break even volume with three machines is 700 units. Hence, the manager would choose two machines and produce 600 units.

2.3 Facility Location and Layout

Questions For Classroom Discussion

Question 1

The present layout is shown in the figure. The manager of the department is intending to interchange the department C and F in the present layout. The handling frequencies between the departments is given. All the departments are of same size and configuration. The material handling cost per unit length travel between departments is same. What will be effect of the interchange of departments C and F in the layout?

A	C	E
B	D	F

Handling Frequency or Trip Matrix

From \ To	A	B	C	D	E	F
A	-------	0	90	160	50	0
B	-------	-------	70	0	100	130
C	-------	-------	-------	20	0	0
D	-------	-------	-------	-------	180	10
E	-------	-------	-------	-------	-------	40
F	-------	-------	-------	-------	-------	-------

Question 2

A defense contractor is evaluating its machine shops current process layout. The figure below shows the current layout and table shows the trip matrix for the facility. Health and Safety regulations require departments E and F to remain at their current positions.

E	B	F
A	C	D

Trip Matrix or Handling

	A	B	C	D	E	F
A	-----	8	3	-----	9	5
B	-----	-----	-----	3	-----	-----
C	-----	-----	-----	-----	8	9
D	-----	-----	-----	-----	-----	3
E	-----	-----	-----	-----	-----	3
F	-----	-----	-----	-----	-----	-----

Question 3 **Home Work - Previous Year Question (December 2008, Q2(d), 6 Mark)**

Mr. X is considering an interchange of departments B and C in the present layout. The present layout and the handling frequencies between the departments are given. What would be the effect of interchange assuming that the departments are of the same size? Also assume that the material handling cost per unit length travel between departments is same.

A	C	E
B	D	F

Handling frequency or Trip Matrix

	A	B	C	D	E	F
A	-----	20	70	0	40	0
B	-----	-----	50	200	0	10

C	-----	-----	-----	30	120	40
D	-----	-----	-----	-----	50	220
E	-----	-----	-----	-----	-----	30
F	-----	-----	-----	-----	-----	-----

Question 4 **Home Work – Previous Year Question (December 10, Q3(a), 6 Mark)**

Given below is the existing process layout of a factory manufacturing toys :

C	B	D
E	A	F

The following table gives the trip matrix for the unit. Arrive at an improved layout using the Load Distance matrix assuming that E and F should remain at their current positions :

Trip Matrix or Handling Frequency

	A	B	C	D	E	F
A	-----	-----	2	2	1	-----
B	-----	-----	1	-----	6	5
C	-----	-----	-----	1	4	-----
D	-----	-----	-----	-----	-----	5
E	-----	-----	-----	-----	-----	1
F	-----	-----	-----	-----	-----	-----

Illustration From Study Material

Illustration 9:

Suppose, an E-Commerce company wants to open Central order fulfilment center in Kolkata South in West Bengal. The possible locations are say L_1, L_2, and L_3. The company form a group of experts. The team identifies say 6 actors such as F_1, F_2, F_3, and F_4 to evaluate L_1 to L_3.

Answer

This situation can be solved using Factor Rating Method. The steps are:

In the first stage the expert team needs to give weightage to the factors. This can be done in many ways. In the following one simple way is explained.

A possible approach:

Suppose, the experts rate each factor on a scale 1 to 5 (1: least important and 5: Most important)

Factor	**Rating**						
	E-1	E-2	E-3	E-4	E-5	**Row**	**Weight**
F_1	4	3	4	4	3	18	18/68
F_2	5	5	5	5	4	24	24/68
F_3	3	4	4	3	5	19	19/68
F_4	2	1	2	1	1	7	7/68
						68	

There may be other ways (e.g., AHP method). Let us now come back to our problem. Let us assume the factors are following weights.

Factors	**Weight**
F_1	0.3
F_2	0.2
F_3	0.1
F_4	0.4
Total	1.0

The experts are requested to rate each of the location alternatives with respect to the factors, e.g., 10: Most beneficial and 1: Least beneficial

Factors	**Alternatives**		
	L_1	L_2	L_3
F_1	10	9	7
F_2	7	3	10
F_3	7	5	10
F_4	6	8	5

So the complete table becomes

Factors	**Weight**	**Alternatives**		
		L_1	L_2	L_3
F_1	0.3	10	9	7
F_2	0.2	7	3	10
F_3	0.1	7	5	10
F_4	0.4	6	8	5
	Best Location	**7.5**	7	7.1

Example of calculation

for L_1 : $0.3 \times 10 + 0.2 \times 7 + 0.4 \times 6 = 3 + 1.4 + 0.7 + 2.4 = 7.5$

As per the weighted score Location L_1 is the best location

Illustration 10:

Suppose, XYZ Ltd wants to open a retail shop in Kolkata, West Bengal.

It first selects the 4 locations such as L_1, L_2, L_3 and L_4. The coordinates of the locations (i.e., latitudes and longitudes) and volume of customers (i.e., average number of customers in a day in '000) are given in the following table

		Coordinates	
Location	Volume	X	Y
L_1	200	30	100
L_2	100	90	120
L_3	100	130	130
L_4	200	60	40

Find out the best location using Center of Gravity (COG) method.

Answer

Loc	**V_1**	**x_i**	**y_i**	**v_i x_i**	**v_i y_i**
L_1	200	30	100	6000	20000
L_2	100	90	120	9000	12000
L_3	100	130	130	13000	13000
L_4	200	60	40	12000	8000
	600		Total	40,000	53,000

Therefore, $SV_i = 600$; $Sv_ix_i = 40000$

$Sv_iy_i = 53000$

COG location is given by (X, Y)

$$X = \frac{\sum V_i X_i}{\sum V_i} = 40000/600 = 200/3$$

$$X = \frac{\sum V_i Y_i}{\sum V_i} = 53000/600 = 265/3$$

Illustration 11:

The present layout is shown in the figure. The manager of the department is intending to interchange the departments C and F in the present layout. The handling frequencies between the departments is given. All the departments are of the same size and configuration. The material handling cost per unit length travel between departments is same. What will be the effect of interchange of departments C and F in the layout?

A	C	E
B	D	F

From / To	A	B	C	D	E	F
A	-	0	90	160	50	0
B	-	-	70	0	100	130
C	-	-		20	0	0
D	-	-	-	_	180	10
E	-	-	-	-	-	40
F	-	-	-	-	-	-

Answer

The distance matrix of the present layout :

From / To	**A**	**B**	**C**	**D**	**E**	**F**
A		1	1	2	2	3
B			2	1	3	2

C				1	1	2
D					2	1
E						1
F						–

(ii) Computation of total cost matrix (combining the inter departmental material handling frequencies and distance matrix.

From / To	A	B	C	D	E	F	Total
A		0	90	320	100	0	510
B			140	0	300	260	700
C				20	0	0	20
D					360	10	370
E						40	40
F							–
Total							1,640

If the departments are interchanged, the layout will be represented as shown below.

A	F	E
B	D	C

The distance matrix and the cost matrix are represented as shown.

From / To	A	B	C	D	E	F
A		1	3	2	2	1
B			2	1	3	2
C				1	1	2
D					2	1
E						1
F						

Total cost matrix for the modified layout.

From / To	A	B	C	D	E	F	Total
A	–	0	270	320	100	0	690
B			140	0	300	260	700
C				20	0	0	20
D					360	10	370
E						40	40
F							–
Total							1,820

The interchange of departments C and F increases the total material handling cost. Thus, it is not a desirable modification.

Illustration 12:

A defence contractor is evaluating its machine shops current process layout. The figure below shows the current layout and the table shows the trip matrix for the facility. Health and safety regulations require departments E and F to remain at their current positions.

E	B	F
A	C	D

Current Layout

From / To	A	B	C	D	E	F
A		8	3		9	5
B		-		3		
C			-		8	9
D		-				3
E					-	3
F						-

Can layout be improved? Also evaluate using load distance (ld) score.

Answer

Keep the departments E and F at the current locations. From the Trip Matrix, C is having maximum no. of trips from E&F. So C must be as close as possible to both E and F, put C between them. Place A directly south of E, and B next to A. All of the heavy traffic concerns have been accommodated. Department D is located in the remaining place. The proposed layout is shown in figure below. The load distance (ld) scores for the existing and proposed layout are shown below. As Id score for proposed layout is less, the proposed layout indicates improvement over existing.

E	C	F
A	B	D

Comparative Analysis: Current and Proposed Layout:

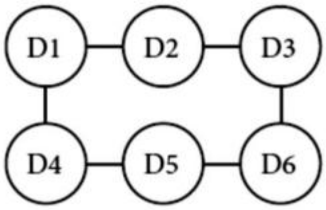
Dept. Pair	**No. of Trips (1)**	**Existing plan**		**Proposed plan**	
		Distance (2)	**Load × Distance (1 × 2)**	**Distance (3)**	**Load × Distance (1 × 3)**
A-B	8	2	16	1	8
A-C	3	1	3	2	6
A-E	9	1	9	1	9
A-F	5	3	15	3	15
B-D	3	2	6	1	3
C- E	8	2	16	1	8
C-F	9	2	18	1	9
D-F	3	1	3	1	3
E-F	3	2	6	2	6
Total			**92**		**67**

As 'ld' score of the proposed layout is lower than the existing one, there is an improvement in the new layout.

Illustration 13:

Suppose a hospital has 6 major departments namely D1, D2, D3, D4, D5 and D6. The initial layout of the hospital is given below.

Initial Layout



The average traffic movement to and from each department is given in the following table

Table – Average traffic flow (Direct)

	D1	D2	D3	D4	D5	D6
D1	–	10	20	0	5	6
D2	8	–	6	10	0	2
D3	10	6	–	20	7	8
D4	0	25	5	–	10	3
D5	15	10	1	20	–	6
D6	0	6	0	3	4	–

The hospital wants to find out an optimum layout.

Answer

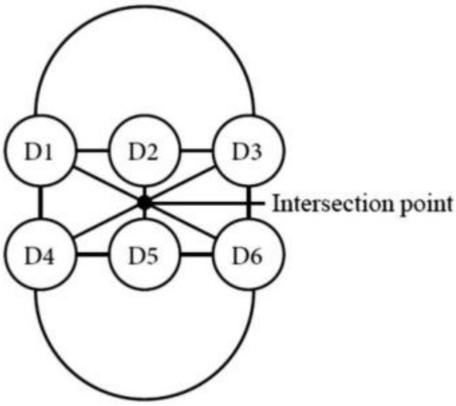
We notice quite obviously that from Di to Di (i = 1, 2….6), there is no movement.
From D2 to D1, the average movement is 10 (circle) and from D1 to D2 the average movement is 8 (circle)
Therefore, the combined average traffic movement from D1 to D2 is = (10 + 8) = 18
Let us now take another pair, e.g., D4 and D2

Movement	**Avg traffic**
D4 → D2	10 (red circle)
D2 → D4	25 (Green circle)

Therefore, the combined average traffic movement is 35. Proceeding in the same way, we get the combined average traffic movement for all pairs as follows:

	D1	**D2**	**D3**	**D4**	**D5**	**D6**
D1	–	18	30	0	20	6
D2		–	12	35	10	8
D3			–	25	8	8
D4				–	30	7
D5					–	10
D6						–

Let us now draw the initial layout again.



Adjacent Pairs	**Non-adjacent Pairs**
D1 & D2	D1 & D3
D2 & D3	D1 & D6
D3 & D6	D3 & D4
D6 & D5	D4 & D6
D5 & D4	
D2 & D5	
D1 & D4	
D1 & D5	
D3 & D5	
D2 & D4	
D2 & D6	

Let us now concentrate on the non-adjacent pairs

Non-adjacent Pair	Distance
D1 & D3	(D1 → D2; D2 → D3) D1 → D3 : 2 nodal points Hence, distance is 2
D1 & D6	D1 → D6 = D1 → P & P → D6 Distance = 2
D3 & D4	D3 → D4 = D3 → P & P → D4 Distance = 2
D4 & D6	D4 → D6 = D4 → D5 & D5 → D6 Distance = 2

The combined average traffic movement between any two non-adjacent nodes is called the load distance. Our objective is to reduce the load distance.

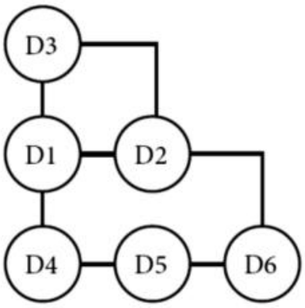
Non-adjacent Pair	Distance
D1 & D3 D3 & D4 D1 & D6 D4 & D6	30 × 2 = 60 25 × 2 = 50 6 × 2 = 12 7 × 2 = 14
	Total = 136

Note that for getting the load values, please refer table (Solution).

To meet our objective, we find the highest load distance, i.e., 60. Therefore, we need to rearrange the nodes.

We notice that from D1 to D3 and back, the highest traffic is involved. Therefore, we need to rearrange their positions to make them adjacent as follows:

First rearrangement

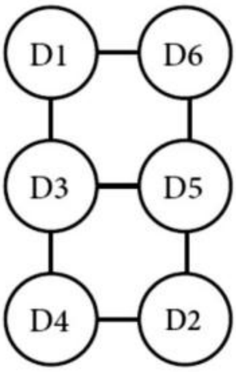


The revised non-adjacent pairs and load distance calculation is given below

Non-adjacent Pair	distance	Load distance
D4 & D6	2	14
D1 & D6	2	12
D3 & D6	2	16
D3 & D5	2	16
D3 & D4	2	50
		108

We notice that there is an improvement. However, now the pair of D3 and D4 creates the problem. Therefore, we need to make them adjacent through rearrangement as follows:

2nd Arrangement



The revised non-adjacent pairs and load distance (after second arrangement) is given below

Non-adjacent Pair	**Load**	**distance**	**load-distance**
D_1D_4	2	0	0
D_6D_2	2	8	16
D_1D_2	2	18	36
D_6D_4	2	7	14
			66

Through trial and error approach we arrive at a considerable improvement. Therefore, the above layout (2nd Arrangement) is the acceptable one.

2.4 Resource Aggregate Planning

Questions For Classroom Discussion

Question 1

ABC company has developed a forecast for the group of items that has the following demand pattern.

Quarter	Demand	Cumulative Demand
1	270	270
2	220	490
3	470	960
4	670	1630
5	450	2080
6	270	2350
7	200	2550
8	370	2920

The firm estimates, it costs Rs 150 per unit to increase production rate, Rs 200 per unit to decrease the production rate. Rs 50 per unit to carry the items in the inventory and Rs 100 per unit, if sub contracted. Compare the costs of the pure strategies?

1) Increase or decrease production
2) Constant Level of production or Inventory carrying strategy
3) Sub Contracting

Illustration From Study Material

Illustration 14:

ABC. Co. has developed a forecast for the group of items that has the following demand pattern

Quarter	Demand	Cumulative demand
1	270	270
2	220	490
3	470	960
4	670	1630
5	450	2080
6	270	2350
7	200	2550
8	370	2920

The firm estimates that it costs Rs. 150 per unit to increase production rate Rs. 200 per unit to decrease the production rate, Rs. 50 per unit per quarter to carry the items in inventory and Rs.100 per unit if subcontracted. Compare the costs of the pure strategies.

Answer

Different pure strategies are

Plan I In this pure strategy, the actual demand is met by varying the work force size. This means that during the period of low demand, the company must fire the workers and during the period of high demand the company must hire workers. These two steps involve associated costs. In this strategy, the production units will be equal to the demand and values in each period. The cost of the plan is computed in the table below,

Quarter	**Demand**	**Cost of increasing Production level (Rs.)**	**Cost of decreasing Production level (Rs.)**	**Total cost of plan (Rs.)**
1	270	—	—	—
2	220	—	50 × 200 = 10,000	10,000
3	470	250 × 150 = 37,500	—	37,500
4	670	200 × 150 = 30,000	—	30,000
5	450	—	220 × 200 = 44,000	44,000
6	270	—	180 × 200 = 36,000	36,000
7	200	—	70 × 200 = 14,000	14,000
8	370	170 × 150 = 25,500	—	25,500
	Total			1,97,000

Plan II In this plan, the company computes the average demand and sets its production capacity to this average demand. This results in excess of units in some periods and also shortage of units during some other periods. The excess units will be carried as inventory for future use and shortage of units can be fulfilled using future inventory. The cost of the plan II is computed in the table below. The plan incurs a maximum shortage of 255 units during quarter 5. The firm might decide to carry 255 units from the beginning of period 1 to avoid shortage. The total cost of the plan is Rs. 96,500.

Quarter	**Demand forecast**	**Cumulative demand**	**Production level = Av. Demand = 2920 ÷ 8**	**Cumu. prod. level**	**Inventory = (Cum. Production – Cum. Demand)**	**Adjusted inventory with 255 at beginning of period 1**	**Cost of holding inventory (Rs.)**
1	270	270	365	365	95	350	17,500
2	220	490	365	730	240	495	24,750
3	470	960	365	1095	135	390	19,500
4	670	1630	365	1460	-170	85	4,250

5	450	2080	365	1825	-255	0	0
6	270	2350	365	2190	-160	95	4,750
7	200	2550	365	2555	5	260	13,000
8	370	2920	365	2920	0	255	12,750
	Total						96,500

Plan III

Normal Production Capacity is assumed to be 200 units i.e. Minimum of the demand values. The additional demand other than the normal capacity is met by subcontracting. The cost of the plan III amounts to Rs. 1,32,000 as shown in table below.

Quarter	**Demand forecast**	**Production units**	**Subcontract units**	**Incremental cost @ Rs. 100/units**
1	270	200	70	70 × 100 = 7,000
2	220	200	20	20 × 100 = 2,000
3	470	200	270	270 × 100 = 27,000
4	670	200	470	470 × 100 = 47,000
5	450	200	250	250 × 100 = 25,000
6	270	200	70	70 × 100 = 7,000
7	200	200	0	0
8	370	200	170	170 × 100 = 17,000
			Total	= 1,32,000

The total cost of pure strategies is given below. On observation Plan II (Changing inventory levels) has the least cost.

Plan	**Total cost (Rs.)**
Plan I	1,97,000
Plan II	96,500
Plan III	1,32,000

Illustration 15:

A firm has developed the following forecast (units) for an item which has a demand influence by seasonal factors.

Month	Forecasted Demand	Production Days
Jan	220	22
Feb	90	18
Mar	210	21
Apr	396	22
May	616	22
Jun	700	20
Jul	378	21
Aug	220	22
Sep	200	20
Oct	115	23
Nov	95	19
Dec	260	20

(a) Prepare a chart showing the daily demand requirements.

(b) Determine the production rate required to meet average demand.

(c) Determine the monthly inventory balance required to follow a plan with:

1. Constant workforce
2. No idle time or overtime
3. No Backorder

4. No use of Sub-Contractor
5. No capacity adjustment

(d) The firm has determined that to follow a plan of meeting demand by varying the size of the workforce strategy

Put result in hiring and lay-off cast estimated at Rs.12000. If the unit cost is Rs.100 each to produce, carrying cost per year are 20% of the average inventory value and storage cost (based upon maximum inventory) are Rs.0.90 per unit which plan results in the lower cost, varying inventory or varying employment? [Where Plan 1 indicates varying inventory and Plan 2 indicates varying Employment]

(e) Suppose the firm wishes to investigate two other plans (alternatives). A third plan is to produce at a rate of 10 units per day and sub-contract the additional requirements at a delivered cost of Rs.107 per unit.

Any accumulated inventory is carried forward at a 20% carrying cost (No extra Storage cost).

The Fourth Plan is to produce at a steady rate of 10 units per day and use overtime to meet the additional requirement at a premium of Rs.10 per unit. Accumulated inventory is again carried forward at a 20% cost.

(f) Compare 4 plans given in Question (d) and (e) and comment which plan gives the minimum cost.

Answer

Chart of Production Requirement

Month	Forecasted Demand	Production Days	Demand/Day	Cumulative Production Days	Cumulative Demand
Jan	220	22	10	22	220
Feb	90	18	5	40	310
Mar	210	21	10	61	520
Apr	396	22	18	83	916
May	616	22	28	105	1532
Jun	700	20	35	125	2232
Jul	378	21	18	146	2610
Aug	220	22	10	168	2830
Sep	200	20	10	188	3030
Oct	115	23	5	211	3145
Nov	95	19	5	230	3240
Dec	260	20	13	250	3500
Total	3500				

(a) Average Requirement = Total Demand / Total Production Days = 3500/25 = 14units/day

(b) Inventory Balance = ∑Production – ∑Demand

Showing the ending Inventory Balance and Ending Balance with Negative Shortage.

Month	Production at 14/day	Forecasted Demand	Inventory Change	Ending Inventory	Balance Ending Balance adjusted in the month of Jan
Jan	308	220	88	88	654
Feb	252	90	162	250	816
Mar	294	210	84	334	900
Apr	308	396	-88	246	812
May	308	616	-308	-62	504
Jun	280	700	-420	-482	84
Jul	294	378	-84	-566	0
Aug	308	220	88	-478	88
Sep	280	200	80	-398	168
Oct	322	115	207	-191	375

Nov	266	95	171	-20	546
Dec	280	260	20	0	566

(c) Maximum Inventory required in storage = 900 units (in the above table Column 6)

Average Inventory Balance = 460 units

Solution to Plan 1 (Varying Inventory):

Inventory Cost = Carrying Cost + Storage Cost

Carrying Cost = 0.20 × 460×100 = 9200

Storage Cost = 900 × 0.90 = 810

Inventory Cost = Rs.10010

Solution to Plan 2 (Varying Employment):

Rs.12000 (Given)

Comparing Plan 1 and Plan 2 we see that Plan 1 is lower.

In case of Plan 3:

it is given that Produce at 10 units per day, vary inventory and sub-contract.

A production rate of 10 units per day exceeds demand only 3 months (Feb, Oct, Nov)

Month	Production at 10/day	Forecasted Demand	Inventory Change
Jan	220	220	0
Feb	**180**	**90**	**90**
Mar	210	210	0
Apr	220	396	-176
May	220	616	-396
Jun	200	700	-500
Jul	210	378	-168
Aug	220	220	0
Sep	200	200	0
Oct	**230**	**115**	**115**
Nov	**190**	**95**	**95**
Dec	200	260	-60

The Inventory Accumulated During these Years must be carried at a cost of (20%) (Rs.100) /12 Months = Rs.1.67 per unit month. Units are Carried until they can be used to help meet demand in a subsequent month

Assume, an equilibrium condition where the excess production from OCT and NOV (150 Units) is on hand JAN

Month	Demand	Production at 10/day	Inventory to carry	Inventory carried until	No. of Months	Cost at $1.67 per unit month
Initial			150	150 units to April	3	750
Feb	90	180	90	26 units to April	2	87
				64 units to May	3	320
Oct	115	230	115	60 units to Dec	2	200
				55 units to Year End	3	275
Nov	95	190	95	95 units to Year End	2	317
					Total	1952

Therefore, Inventory Cost from above = Rs.1952

Calculating Marginal Cost of Sub-contracting:

The marginal cost of sub-contracting

Number of units = Demand – Production = 3500 – (10 × 250) = 1000 units for sub-contracting

Therefore, Cost per unit = Rs.107 – Rs.100 = Rs.7 per unit

Therefore, Marginal Cost = 1000 units × Rs.7 per unit = Rs.7000

The total Cost of Plan 3 = Inventory Cost + Sub-contracting cost = 1952 + 7000 = Rs.8952

Plan 4:

This plan differs from plan 3 only in the marginal cost which is now due to overtime rather than sub-contracting.

So, Inventory cost (same as plan 3) i.e., Rs.1952 and Marginal cost of Overtime = 1000 units × rate of Rs.10 per unit = Rs.10,000

Therefore, total cost of Plan 4 = Rs.10,000 + Rs.1952 = Rs.11952

Table: Comparison of Plans

Plan	Strategy	Cost
Plan 1	Pure Strategy (Vary Inventory)	Rs.10010
Plan 2	Pure Strategy (Vary Employment)	Rs.12000
Plan 3	Mixed Strategy (Sub-contract and Vary Inventory)	Rs.8952
Plan 4	Mixed Strategy (Overtime and Vary Inventory)	Rs.11952

2.7 Economic Batch Quantity

Questions For Classroom Discussion

Question 1

XYZ company carries a wide assortment of items for its customers. One Item 'A' is very popular. Desires of keeping its inventory under control, a decision is taken to order only the optimal economic quantity, for this item, each time. Given Annual Demand, 160,000 units. Price per unit: Rs 20, Carrying cost Rs 4 per unit. Cost of order Rs 50.

Develop the following and from the table determine the optimal order quantity.

Number of Order	1	10	20	40	80	100
Order Quantity						
Average Inventory						
Order Cost @ 50						
Carrying Cost @ 4						
Total inventory Management Cost						

Question 2

The monthly requirement of raw material for a company is 3000 units. The carrying cost is estimated to be 20% of the purchase price per unit, in addition to Rs 2 per unit. The purchase price of raw material is Rs 20 per unit. The ordering cost is Rs 25 per order.

Q.1 You are required to find EOQ.

Q.2 What is the total cost when the company gets a concession of 5% on the purchase price if it orders 3000 units or more but less than 6000 units per month

Q.3 What happens when the company gets a concession of 10% on the purchase price when it orders 6,000 units or more?

Q.4 Which of the above three ways of orders the company should adopt?

Question 3

M/s Kobo Bearings Ltd., is committed to supply 24,000 bearings per annum to M/s Deluxe Fans on a steady daily basis. It is estimated that it costs 10 paisa as inventory holding cost per bearing per month and that the setup cost per run of bearing manufacture is Rs 324.

Q.1 What is the optimum run size for bearing manufacture?

Q.2 What should be the interval between the consecutive optimum runs?

Q.3 Find out the minimum inventory management cost.

Question 4

Shravan Tubes Ltd. are the manufacturers of picture tubes of T.V. The following are the details of their operation during 2001:

- Average monthly market demand 2,000 tubes
- Ordering cost Rs 100 per order.
- Inventory carrying cost 20% per annum.
- Cost of tubes Rs 500 per tube.
- Normal usage 100 tubes per week.
- Minimum usage 50 tubes per week.
- Maximum usage 200 tubes per week.
- Lead time to supply 6 – 8 weeks

Find

Q.1 Economic order quantity. If the supplier is willing to supply quarterly 1,500 units at a discount of 5%, is it worth accepting?

Q.2 Maximum level of stock.

Q.3 Minimum level of stock.
Q.4 Re-order level of stock.

Illustration From Study Material

Illustration 16:

The monthly requirement of raw material for a company is 3000 units. The carrying cost is estimated to be 20% of the purchase price per unit, in addition to Rs. 2 per unit. The purchase price of raw material is Rs. 20 per unit. The ordering cost is Rs. 25 per order. (i) You are required to find EOQ.(ii) What is the total cost when the company gets a concession of 5% on the purchase price if it orders 3000 units or more but less than 6000 units per month. (iii) What happens when the company gets a concession of 10% on the purchase price when it orders 6,000 units or more? (iv) Which of the above three ways of orders the company should adopt?

Answer:

We are given that,

A = Annual demand = 3,000 × 12 = 36,000 units per annum ; S = Ordering Cost = Rs. 25;

C = Inventory carrying cost = 2 + 20% of Rs. 20 = 2 + 4 = Rs. 6

(i) $\text{EOQ} = \sqrt{\frac{2AS}{C}} = \sqrt{\frac{2 \times 36000 \times 25}{6}} = \sqrt{30,00,000} = 548\text{units(approx.)}$

Total cost = Ordering Cost + Cost of purchasing the material + Storage cost

= (36,000 / 548) × 25 + (36,000 × 20) + (548/2) × 6 [∵ Storage cost = Average Inventory × Inventory carrying cost

= Rs. 1642.33 + 7,20,000+ 1,644 = Rs. 7,23,286. $= \frac{\text{EOQ}}{2} \times 6]$

(ii) When the company has an option to order between 3000 and 6000 units, the EOQ should be calculated with a reduction in price by 5% (due to concession); The purchase price = 95% of Rs. 20 = Rs. 19.

A = 36,000 units per annum; S = Rs. 25; C = 2 + 20% of 19 = 2 + 3.80 = Rs. 5.80

$\text{EOQ} = \sqrt{\frac{2 \times 36000 \times 25}{5.80}} = \sqrt{\frac{1,80,000}{5.80}} = 557\text{units(approx)}$

Total cost = (36,000/557) × 25 + (36,000 × 19) + (557/2) × 5.80

= Rs. (1,615.79 + 6,84,000 + 1,615.30) = Rs. 6,87,231.09

For monthly order quantity being 3000 units or more but less than 6000 units

EOQ = 557 units

No. of orders per year $= \frac{\text{Yearlydemand}}{\text{EOQ}} = \frac{36000}{557} = \text{N(Let)}$

No. of orders per month $= \frac{N}{12} = \frac{36000/557}{12} = 5.385 = 6(\text{Say}) = N^*$

Quantity to be orderd per month = N* × EOQ = 6 ×557 = 3342 units

This quantity lies in the range of 3000 to 6000 units

Hence the EOQ (557 units) can be considered to be a feasible quantity for availing 5% discount on Purchase Price.

(iii) When the company orders more than 6,000 units purchase price = 90% of Rs. 20 (because 10% concession)

= Rs. 18; A = 36,000 units per annum; S = Rs. 25; C = 2 + 20% of Rs. 18

= 2 + 3.60 = 5.60

$\text{EOQ} = \sqrt{\frac{2AS}{C}} = \sqrt{\frac{2 \times 36000 \times 25}{5.60}} = 567 \text{ units app.}$

For monthly order quantity more than or equal to 6000 units

EOQ = 567 units

No of orders per month $= \frac{36000/567}{12} = 5.29 = 6(\text{Say}) = N^*$

Qty. to be ordered per month = N* × EOQ = 6×567 = 3402 units

This quantity does not lie in the range of 6000 or more units.

Hence the EOQ (567 units) can not be considered as feasible quantity for availing 10% discount on Purchase Price.

To understand the effect of 10% on Total Cost, we consider the minimum value of price break quantity of this range i.e. 6000 units to be the optimum order quantity and calculate.

Total Cost as follows —

TC = Ordering Cost + Cost of Purchasing the material + Storage Cost

$$= \frac{36000}{6000} \times 25 + 36000 \times 18 + \frac{6000}{2} \times 5.60$$

= 150 + 648000 + 16800 = Rs. 6,64,950

Hence the total cost will be minimum (Rs. 6,64,950) if orders are placed in lot size of 6000 units.

Illustration 17:

M/s. Tubes Ltd. are the manufacturers of picture tubes of T.V. The following are the details of their operation during 2001:

Average monthly market demand	2,000 tubes
Ordering cost	Rs. 100 per order
Inventory carrying cost	20% per annum
Cost of tubes	Rs. 500 per tube
Normal usage	100 tubes per week
Minimum usage	50 tubes per week
Maximum usage	200 tubes per week
Lead time to supply	6 – 8 weeks

Compute from the above:

(1) Economic order quantity. If the supplier is willing to supply quarterly 1,500 units at a discount of 5%, is it worth accepting?

(2) Maximum level of stock.

(3) Minimum level of stock.

(4) Re-order level of stock.

Answer

(1) Economic Order Quantity:

Annual usage of tubes (A) = Normal usage per week × 52 weeks
= 100 tubes × 52 weeks
= 5,200 tubes.

Ordering cost per order (S) = Rs. 100.

Inventory carrying cost per unit per annum (C) = 20% of Rs. 500 = Rs. 100.

$$\text{EOQ} = \sqrt{\frac{2AS}{C}} = \sqrt{\frac{2 \times 200\text{units} \times \bullet 100}{\bullet 100}} = 102 \text{ units (approx.)}$$

(A) Evaluation of order size of 1,500 units at 5% discount

$$\text{No. of orders} = \frac{5{,}200 \text{ uits}}{1{,}500 \text{ units}} = 3.46 \text{ or } 4$$ (in case of a fraction, the next whole number is considered).

	Rs.
Ordering cost (No. of order per year at Rs. 100 per order)	400
Carrying cost of average inventory:	
$\frac{1{,}500 \text{ uits}}{2} \times \text{Rs.}(50\text{less } 5\%) \times \frac{20}{100}$	71,250
Total annual cost (excluding item cost)	71,650
(B) Annual cost if EOQ (102 units) is adopted :	Rs.
Ordering cost: 5,200 ÷ 102 or 51 orders per year at Rs.100 per order	5,100

Carrying cost of average inventory $\frac{102 \text{ units}}{2} \times \text{Rs.}500 \times \frac{20}{100}$	5,100
Total annual cost (excluding item cost)	10,200

Increase in annual cost by adopting (A) above : Rs. (71,650 – 10,200) = Rs. 61,450.

Amount of quantity discount: 5% × Rs. 500 × 5,200, units = Rs. 1,30,000.

Since the amount of quantity discount (Rs. 1,30,000) is more than the increase in total annual cost (Rs. 61,450), it is advisable to accept the offer. This will result in a saving of Rs. (1,30,000 - 61,450) or Rs. 68,550 p.a. in inventory cost.

(2) Maximum Level of Stock:

= Re-order level + Re-order quantity – (Minimum usage × Minimum delivery period) = 1,600 units + 102 units – (50 units × 6 weeks) = 1,402 units.

[Assume that the Reorder quantity is supplied as soon as the Reorder level is reached]

(3) Minimum Level of Stock:

= Re-order level – (Normal usage × Normal delivery period) [see Note] = 1,600 units – (100 units × 7 weeks)

= 900 units. Note: Normal delivery period is taken to be the average delivery period.

(4) Re-order Level of Stock:

= Maximum usage × Maximum delivery period = 200 units × 8 weeks = 1,600 units.

Illustration 18:

M/s Kobo Bearings Ltd., is committed to supply 24,000 bearings per annum to M/s Deluxe Fans on a steady daily basis. It is estimated that it costs 10 paisa as inventory holding cost per bearing per month and that the setup cost per run of bearing manufacture is Rs. 324.

(a) What is the optimum run size for bearing manufacture?

(b) What should be the interval between the consecutive optimum runs?

(c) Find out the minimum inventory holding cost.

Answer

(a) Optimum run size or Economic Batch Quantity (EBQ)

$$\sqrt{\frac{2 \times \text{Annual Output} \times \text{Setup cost}}{\text{Annual Cost of Carrying one unit}}} = \sqrt{\frac{2 \times 24000 \times 324}{0.10 \times 12}} = 3600 \text{ units}$$

(b) Interval between two consecutive optimum runs = $\frac{\text{FBQ}}{\text{Monthly Output}} \times 30$

$$= \frac{3600}{24000 \div 12} \times 30 = 54 \text{ Calendar days}$$

(c) Minimum inventory holding cost = Average inventory × Annual carry-ing cost of one unit of inventory

= (3600 ÷ 2) × 0.10×12 = Rs. 2,160.

Illustration 19:

A company planning to manufacture a household cooking range has to decide on the location of the plant. Three locations are being considered viz., Patna, Ranchi, and Dhanbad. The fixed costs of the three location are estimated to be Rs.30 lakh, Rs.50 lakh, and Rs.25 lakh per annum respectively. The variable costs are Rs.300, Rs.200 and Rs.350 per unit respectively.

The expected sales price of the cooking range is Rs.700 per unit Find out:

(i) The range of annual production/sales volume for which each location is most suitable and

(ii) Which one of the three locations is the best location at a production/sales volume of 18,000 units?

Answer

The total cost of the three locations are:

At Total cost = Fixed cost + Variable cost for a volume "X"

Patna => Total cost = 30,00,000 + 300 × X

Ranchi => Total cost = 50,00,000 + 200 × X

Dhanbad => Total cost = 25,00,000 + 350 × X

We can compute and plot the total costs per annum at the three different locations for the various cases of production volume of 5,000, 10,000, 15,000, 20,000 25,000 units.

(i) Patna

Volume (x Units)	5,000	10,000	15,000	20,000	25,000
Fixed Cost (Rs.)	30,00,000	30,00,000	30,00,000	30,00,000	30,00,000
Variable Cost (Rs. 300 x)	300 (5,000)	300 (10,000)	300 (15,000)	300 (20,000)	300 (25,000)
Total Cost (Rs.)*	= Rs.45 lakhs	= Rs.60 lakhs	= Rs.75 lakhs	= Rs.90 lakhs	= Rs.105 lakhs

(ii) Ranchi

Volume (x Units)	5,000	10,000	15,000	20,000	25,000
Fixed Cost (Rs.)	50,00,000	50,00,000	50,00,000	50,00,000	50,00,000
Variable Cost (Rs. 200 x)	200 (5,000)	200 (10,000)	200 (15,000)	200 (20,000)	200 (25,000)
Total Cost (Rs.) *	= Rs.60 lakhs	= Rs.70 lakhs	= Rs.80 lakhs	= Rs.90 lakhs	= Rs.100 lakhs

(iii) Dhanbad

Volume (x Units)	5,000	10,000	15,000	20,000	25,000
Fixed Cost (Rs.)	25,00,000	25,00,000	25,00,000	25,00,000	25,00,000
Variable Cost (Rs. 300x)	350 (5,000)	350 (10,000)	350 (15,000)	350 (20,000)	350 (25,000)
Total Cost (Rs.)*	= Rs.42.5 lakhs	= Rs.60 lakhs	= Rs.77.5 lakhs	= Rs.95 lakhs	= Rs.112.5 lakhs

* In all the above tables, Total Cost = Fixed Cost + Variable Cost

If the volume distribution be as follows:

	Up to 10,000 units	**Between 10,000 units to 20,000 units**	**Above 20,000 units**
Favourable Location	Dhanbad	Patna	Ranchi

For a volume of 18000 units favourable location is Patna which can be substantiated by the followings calculations

of Total Cost :-

Patna => 30,00,000 + 300 × 18,000 = Rs.84 lakhs

Ranchi => 50,00,000 + 200 × 18,000 = Rs.86 lakhs

Dhanbad => 25,00,000 + 350 × 18,000 = Rs.88 lakhs.

Illustration 20:

Monthly demand for a component is 1000 units. Setting-up cost per batch is Rs. 120. Cost of manufacture per unit is Rs. 20. Rate of interest may be considered at 10% p.a. Calculate the EBQ.

Answer:

Calculation of EBQ:

$$\text{EBQ} = \sqrt{\frac{2 \times \text{Annual Demand} \times \text{Setup cost}}{\text{Unit Cost} \times \text{Inventory Carrying cost per unit per year}}} = \sqrt{\frac{2 \times 12 \times 1000 \times 120}{(0.10 \times 20)}} = 1200 \text{ units}$$

Illustration 21:

Based on the following data on the exports of an item by a company during the various years fit a straight line, (for the time being, assume that a straight line gives a good fit). Give a forecast for the years 2013 and 2014.

Year	No. of items ('000)
2004	13
2005	20
2006	20
2007	28
2008	30
2009	32
2010	33
2011	38
2012	43

Answer:

We can call the years as 'X' and exports as 'Y. In order to use the normal equations for the least square line, we need ΣX, ΣY, ΣXY and ΣX2. If we arrange X in such a way that ΣX = 0, it will simplify our calculations. Therefore, we call the year 2008 as 0, 2007 as -1 and 2009 as + 1 and likewise for the other years in the data.

The rearrangement is shown in the table as follows:

X	Y	X^2	XY
-4	13	16	-52
-3	20	9	-60
-2	20	4	-40
-1	28	1	-28
0	30	0	0
1	32	1	32
2	33	4	66
3	38	9	114
4	43	16	172
ΣX = 0	ΣY = 257	$\Sigma x^2 = 60$	ΣXY - 204

Let the equation of the best fit straight line to the given data be $Y = a_0 + a_1x$

So the normal equations are

$\Sigma Y = a_o N + a_1 \Sigma X$ .... (1)

$\Sigma XY = a_o \Sigma X + a_1 \Sigma X2$ .... (2)

As ΣX = 0, from (1) $\Sigma Y = a_o N$ from (2) $\Sigma XY = a_1 \Sigma X^2$

Therefore, $a_o = \Sigma Y / N = 257 / 9 = 28.56$ [N = No. of years]

$a_1 = \Sigma XY / \Sigma X^2 = 204 / 60 = 3.4$

The equation of a straight line fitting the data is:

Y = 28.56 + 3.4 X

(a) Forecast for 2013, (i.e., X = 5): Y = 28.56 + 3.4 (5) = 45.56 ('000) nos.

(b) Forecast for 2014, (i.e., X = 6): Y = 28.56 + 3.4 (6) = 48.96 ('000) nos.

Illustration 22:

Find the economic order quantity and the reorder point, given

Annual demand (D) = 1000 units

Average daily demand (d) = 1000/365

Ordering Cost (S) = Rs. 5 per order

Holding cost(H) = Rs. 1.25 per unit per year.

Lead time (L) = 5 days
Cost per unit (C) = Rs. 12.50
What quantity should be ordered?

Answer:

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 1000 \times 5}{1.25}} = \sqrt{8000} = 89.44 \text{ units}$$

$$\text{Re-order unit} = dL = \frac{1000}{365} \times 5 = 13.7 \text{ units}$$

$$\text{Total Cost} = DC + \frac{D}{Q} \times S + \frac{Q}{2} \times H = 1000 \times 12.5 + (1000 / 89.44) \times 5 + (89.44 / 2) \times 1.25$$

= Rs. 2611.81

Illustration 23:

Consider an economic order quantity case where annual demand D=1000 units, economic order quantity Q= 200 units , the desired probability of not stocking out P=0.95 , the standard deviation of demand during lead time 6L =25units and lead time = L=15 days. Determine the reorder point. Assume the demand is over a 250 week day year.

Answer:

d = D/no. week days = 1000/250 = 4
Re-order level(R) = dL + z L = 4×15 + 1.64 × 25 = 101

Illustration 24:

Daily demand for a certain product is normally distributed with a mean of 60 and standard deviation of 7. The source of supply is reliable and maintain a constant lead time of six days. The cost of placing the order is Rs.10 and annual holding costs are Rs.0.50 per unit. There are no stock out costs, and unfilled orders are filled as soon as the order arrives. Assume sales occur over the entire 365 days of the year. Find the order quantity and reorder point to satisfy a 95 percent probability of not stocking out during the lead time.

Answer:

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times (60 \times 365)}{0.5}} = \sqrt{876000} = 936 \text{ units}$$

$$\sigma_1 = \sqrt{\sum_{i=1}^{L} \sigma_d^2} = \sqrt{6 \times 7^2} = 17.15$$

Re-order leve(R) = dL + z 6L = 60×6 + 1.64 × 17.15 = 188

Illustration 25:

Fixed -Time period Model with safety stock

Daily demand for a product is 10 units with a standard deviation of 3 units. The review period is 30 days, and lead time is 14 days. Management has set a policy of satisfying 98% of demand from items in stock. At the beginning of this review period, there are 150 units in inventory.

Answer:

$$Q = d(T+L) + z\,\sigma_{T+L} - I = 10\,(30+14) + T\,\sigma_{T+L} - 150 =$$

$$\sigma_{T+L} = \sqrt{\sum_{i+1}^{T+L} \sigma_d^2} = \sqrt{(T+L)\sigma_d^2} = \sqrt{(30+40) \times 3^2} = 19.90$$

T for P = 0.98 is 2.05
Q = 160 – 150 + 2.05×19.9 = 331 units

Illustration 26:

Average Inventory calculation – Fixed order quantity model

Suppose the following item is being managed using a fixed order quantity model with safety stock

Annual Demand (D) = 1000 units

Order quantity (Q) = 300 units

Safety stock (SS) = 40 units

What are the average inventory level and inventory turn for the item?

Answer:

Avg. Inventory = Q/2 + SS = 150 + 40 =190

$$\text{Inventory Turn} = \frac{D}{\frac{Q}{2} + SS} = \frac{1000}{190} = 5.263 \text{ tern per year}$$

Illustration 27:

Average Inventory calculation – Fixed Time period model

Consider the following item that is being managed using a fixed time period model with safety stock

Weekly demand (d) = 50 units

Review cycle (T) = 3 weeks

Safety stock (SS) = 30 units

What are the average inventory level and inventory turn for the item?

Answer:

Avg. Inventory = dT/2 + SS = (50 × 3)/2 + 30 = 105 units

$$\text{Inventory Turn} = \frac{52d}{\text{Avg. Inventory}} = \frac{52 \times 50}{105} = 24.8 \text{ turn per year}$$

Illustration 28:

Price Break Problem

Consider the following case, where

D = 10000 units (annual demand)

S = Rs. 20 to place order

I = 20 percent of cost (annual carrying cost, storage, interest, obsolescence, etc)

C = Cost per unit (according to the order size: order of 0 to 499 units, Rs.5.00 per unit; 500 to 999 units, Rs.4.50 per unit; 1000 and up, Rs.3.90 per unit)

What quantity should be ordered?

Answer:

$$EOQ1 = \sqrt{\frac{2DS}{iC}} = \sqrt{\frac{2 \times 10000 \times 20}{20 \times 5}} = 63.24$$

$$EOQ2 = \sqrt{\frac{2 \times 10000 \times 20}{20 \times 4.5}} = 66.67$$

$$EOQ3 = \sqrt{\frac{2 \times 10000 \times 20}{20 \times 3.9}} = 71.6$$

$$\text{Total Cost1} = DC + \frac{D}{Q} \times S + \frac{Q}{2} \times iC = 56323$$

TC2 = 51000

TC3 = 44585.69

1000 units should be ordered.

Illustration 29:

A product is priced to sell at Rs.100 per unit, and its cost is constant at Rs.70 per unit. Each unsold unit has a salvage value of Rs.20. Demand is expected to range between 35 and 40 units for the period. 35 definitely can be sold and no units over 40 will be sold. The demand probabilities and the associated cumulative probability distribution (P) for this situation fo;;ow.

Number of Units	Demanded Probability of this Demand	Cumulative Probability
35	0.10	0.10
36	0.15	0.25
37	0.25	0.50
38	0.25	0.75
39	0.15	0.90
40	0.10	1.00

How many units should be ordered?

Answer:

The cost of underestimating the demand is loss of profit (Cu) or 100-70=30/unit. The cost of overestimating demand is the loss in occured when the unit must be sold at salvage value(Co)

= 70 – 20 = 50

The optimal prob. Of not being sold

P<= Cu/Co + Cu = 30/30 + 50 = 0.375

From the data, this corresponds to 37th value.

No. of unit sold

Unit demand	**Prob.**	**35**	**36**	**37**	**38**	**39**	**40**
35	0.1	0	50	100	150	200	250
36	0.15	30	0	50	100	150	200
37	0.25	60	30	0	50	100	150
38	0.25	90	60	30	0	50	100
39	0.15	120	90	60	30	0	50
40	0.1	150	120	90	60	30	0
Total	1	75	53	43	53	83	125

Illustration 30:

A company currently has 200 units of a product on hand that it orders every two weeks when the salesperson visits the premises. Demand for the product averages 20 units per day with a standard deviation of 5 units. Lead time for the product to arrive is seven days. Management has a goal of 95 percent probability of not stocking out for this product. The salesperson is due to come in late this afternoon when 180 units are left in stock (assuming that 20 are sold today). How many units should be ordered?

Answer:

S.D = $\sqrt{21\,(5) \times (5)} = 23$

Z = 1.64

q = d×(T + L) + Z×S.D-I

= 20(14 + 7) + 1.64×23-180

= 278 units

Illustration 31:

Solve the ABC analysis of the following table and show graphically taking Percentage of total list of different stock items as x axis and Percentage of total inventory value along y axis

Annual Usage if Inventory by Value

Item Number	Annual Rupee Usage (Rs.)	Percentage of total value (%)
22	95000	40.69
68	75000	32.13
27	25000	10.71
03	15000	6.43
82	13000	5.57
54	7500	3.21
36	1500	0.64
19	800	0.34
23	425	0.18
41	225	0.10
TOTAL	Rs. 233450	100%

Answer:

Classification	Item no.	Annual Rupee Usage	% of total
A	22,68	1,70,000	72.9%
B	27,03,82	53,000	22.7%
C	54,36,19,23,41	10,450	4.5%

4. Application of Operation Research - Production Planning and Control

4.4 Linear Programming Problem

Questions For Classroom Discussion

Question 1

A home resourceful decorator manufacturers two types of lamps, Say A and B. Both Lamps go through two technicians, first a cutter and second a finisher. Lamp A requires 2 hours of the cutter's time and one hour of finishers time. Lamp B requires 1 hour of cutter's time and 2 hour of finishers time each month. The cutter has 104 hours of available time and finisher has 76 hours of available time each month. Profit on the Lamp A is Rs 6 and on lamp B is Rs 11. Formulate Mathematical Model?
(Formulate Linear Programming Model).

Question 2

A manufacturer of furniture makes two products, chairs and tables. Processing of these products is done on two machines A and B. A chair requires 2 hours on machine A and 6 hours on machine B.A table requires 5 hours on machine A and no time on machine B. Machine A has an available time of 16 hours and Machine B has an available time of 30 hour per day. Profit gained by the manufacturer from a chair is Rs 100 and from a table is Rs 500. Formulate the problem into a linear programming problem, the objective is to maximize the total profit.

Question 3

A firm can produce three types of clothes, say A,B,C. Three kinds of wool were required for it, say red wool, green wool and blue wool. One unit length of type A cloth needs 2 yards of red wool and 3 yards blue wool. One unit length of type B cloth needs 3 yards of red wool, 2 yards of green wool and 2 yards of blue wool. One unit of type C cloth needs 5 yards of green wool and 4 yards of blue wool. The firm has only a stock of 8 yards of red wool, 10 yards of green wool and 15 yards of blue wool. It is assumed that the income obtained from one unit length of Type A cloth is Rs 3, of type B cloth is Rs 5 and Type C cloth is Rs 4. Formulate LPP?

Question 4

A company produces two types of cow boy hats. Each hat of the first type requires twice as much labour as the second type. If all hats are of the second type only, the company can produce a total of 500 hats a day. The market limits daily sales of the first and second types to 150 and 250 hats. Assuming the profit per hat is Rs 8 for type 1 and Rs 5 for type 2. Formulate the problem as a linear programming model to determine the number of hats to be produced of each type so as to maximize the profit?

Question 5

An animal feed company must produce at least 200 kg's of a mixture consisting ingredients Coconut oil cake and Sesame Seeds oil cake. Coconut cake costs Rs 3 per kg and Sesame seeds cake cost Rs 8 per Kg. No more than 80 Kg's of coconut oil cake can be used and at least 60 Kg's of Sesame Seeds oil cake must be used. Formulate a mathematical model to the problem.

Question 6

A marketing manager wishes to allocate his annual advertising budget of Rs 2,000,000 in two media vehicles Griha Lakshmi and Vanitha. The unit cost of a message in Media Griha Lakshmi is Rs 100,000 and that of Vanitha is Rs 150,000. Media Griha Lakshmi is a monthly magazine and not more than one insertion is desired in one issue. At least 5 messages should appear in Vanitha. The expected effective audience for a unit message in Griha Lakshmi is 40,000 and that for Vanitha is 55000.Develop a mathematical Model
(Formulate linear programming problem)

Question 7

A manufacturer of a line patent medicines is preparing a production plan for medicines A and B. There are sufficient ingredients available to make 20,000 bottles of A and 40,000 bottles of B but there are only 45000 bottles into which either of the medicines can be put. Furthermore, it takes 3 hours to prepare enough materials to fill 1000 bottles of A, it takes one hour to prepare enough materials to fill 1000 bottles of B and there are 66 hours of available time for the operation. The Profit is Rs 8 per bottles for A and Rs 7 for B. Formulate this problem as a linear programming problem.

Advanced Questions

Question 1 **Advanced Question**

The agricultural research institute suggested to a farmer to spread out at least 4800 Kg, of Special Phosphate Fertilizer and not less than 7200 Kg of a Special Nitrogen fertilizer to raise productivity of crops in his fields. There are two sources for obtaining this, Mixture A and Mixture B. Both of these are available in bags weighing 100 Kg each and they cost Rs 40 and Rs 24 respectively. Mixture A contains Phosphate and Nitrogen equivalent of 20 Kg and 80 Kg respectively while Mixture B contains these ingredients equivalent of 50 Kg each. Formulate LPP in order to determine the number of bags, so that total cost is minimized ?

Different Methods of Solving LPP

1: Graphical Method

2: Simplex Method

3: Transportation Problem

4: Assignment Problem

Question 2 **Advanced Question 6 Mark**

A 24 hour super market has the following minimal requirement of cashiers.

Period	Time of the day	Minimum Number Required
1	3 to 7	7
2	7 to 11	20
3	11 to 15	14
4	15 to 19	20
5	19 to 23	10
6	23 to 03	5

Period 1 follows immediately after period 6. A cashier works eight consecutive hours, starting at the beginning of the six time periods. Determine a daily employee worksheet, which satisfies the requirements with least number of personnel. Formulate the problem as a linear programming problem.

A city hospital has the following minimal daily requirement for nurses:

Period	Time of the day	Number of nurses required
1	6 am to 10 am	2
2	10 am to 2 pm	7
3	2 pm to 6 pm	15
4	6 pm to 10 pm	8
5	10 pm to 2 am	20
6	2 am to 6 am	6

Nurses report to the hospital at the beginning of each period and work for consecutive 8 hours. The hospital wants to determine the minimal number of nurses to be employed so that there will be sufficient number of nurses available for each period. Formulate LPP. Do not solve.

PTP – December 2014 – Set C

Question 3 **Advance Question**

An agriculturist has a 125 acre farm. He produces radish, Green peas and Potato. Whatever he raises is fully sold in the market. He gets Rs 5 for radish, Rs 4 for Green peas and Rs 5 for potato per kg. The average per acre yield is 1500 kg of radish, 1800 kg of green peas and 1200 kg of potato. To produce each 100 kg of radish and green peas and 80 kg of potato, a sum of Rs 12.50 has to be used for manure. Labour required for each acre to raise the crop is 6 man days for radish and potato each and 5 man days for green peas. A total of 500 man days of labor at a rate of Rs 40 per day available. Formulate LPP to maximize profit?

Question 4 **Advanced Question**

The marketing department of Everest company has collected information on the problem of advertising for its products.

This relates to the advertising media available, the number of families expected to be reached with each alternative, cost per advertisement, the maximum availability of each medium and expected exposure of each one .

The information is given below.

Advertising Media	Number of families Expected to cover	Cost per Advertisement	Maximum Availability	Expected Exposure
TV (30 Seconds)	3000	8000	8	80
Radio (15 Seconds)	7000	3000	30	20
Sunday Edition of Daily (Quarter Page)	5000	4000	4	50
Magazine (1 Page)	2000	3000	2	60

Other Information:

1) Advertising budget is 70,000
2) At least 40,000 families should be covered.
3) At least 2 insertions be given in Sunday edition of daily but no more than 4 ads should be given on the TV.

Draft this as a linear programming model. The company's objective is to maximize the expected exposure.

Question 5 **Advanced Question**

A manufacturing firm needs 5 component parts. Due to inadequate resources, the firm is unable to manufacture all its requirements. Thus, the management is interested in determining as to how many, if any, units of each component should be purchased from outside and how many should be produced internally. The relevant data are given here.

Components	Milling Hours	Assembly Hours	Per Unit Testing Hours	Total Requirements in Units	Price Per unit in the market	Per unit Cost
C1	4	1	15	20	48	30
C2	3	3	2	50	80	52
C3	1	1	0	45	24	18
C4	3	1	0.5	70	42	31
C5	2	0	0.5	40	28	16

Resources available are as follows
Milling Hours : 300 Hours
Assembly hours : 160 Hours
Testing Hours : 150 Hours

Formulate this as an LPP, taking objective function as maximization of saving by producing the components internally.

Question 6 **Advanced Question**

A leading chartered accountant is attempting to determine a best investment portfolio and is considering 6 alternative investment proposals. The following table indicates point estimates for the price per share, annual growth rate in the price per share, the annual dividend per share and a measure of risk associated with each investment.

Shares Under Consideration	A	B	C	D	E	F
Current Share Price	80	100	160	120	150	200
Projected Annual Growth Rate	0.08 or (8%)	0.07 or (7%)	0.10 or (10%)	0.12 or (12%)	0.09 or (9%)	0.15 or (15%)
Projected Annual Dividend Per Share	4	4.5	7.5	5.5	5.75	0
Projected Risk in Return	0.05	0.03	0.10	0.20	0.06	0.08

1. Total amount available for investment is Rs 2,500,000 and the following conditions are to be satisfied
2. The maximum rupee amount to be invested in alternative F is 250,000
3. No more than 500,000 should be invested in alternatives A and B combined
4. Total weighted risk should not be greater than 0.10 ,where

$$\text{Total weighted risk} = \frac{\text{(Amount invested in alternative x Risk of alternative)}}{\text{Total investment in alternative}}$$

5. For the sake of diversity, at least 100 share is each stock should be purchased
6. At least 10 percentage of total investment should be in alternatives A and B combined.
7. Dividends for the year should be at least Rs 10,000.

Illustration From Study Material

Illustration 1:

A Chemical Company produces two compounds A and B. The following table gives the units of ingredients C and D per kg of compounds A and B as well as minimum requirements of C and D and costs/kg of A and B. Write drawn the problem mathematically for minimisation of cost.

		Table Compound		Minimum requirement
		A	B	
Ingredient	C	1	2	80
	D	3	1	75
Cost per kg.		4	6	

Answer:

Let x_1 be the no. of units of A

Let x_2 be the no. of units of B

Objective function: Min.Z = $4x_1 + 6x_2$

Subject to Constraints:

$x_1+2x_2 \geq 8$ (Constraint on requirement of ingradient C)

$3x_1+x_2 \geq 75$ (Constraint on requirement of ingradient D)

And $x_1, x_2 \geq 0$ (No negativity constraint)

Illustration 2:

A pension fund manager is considering investing in two shares A and B. It is estimated that:

(i) Share A will earn a dividend of 12% per annum and share B 4% per annum.

(ii) Growth in the market value in one year of share A will be 10 paise per Rs.1 invested and in B 40 paise per Rs.1 invested.

He requires investing the minimum total sum which will give:

Dividend income of at least Rs.600 per annum and growth in one year of at least Rs.1,000 on the initial investment.

You are required to:

State the mathematical formulation of the problem which will facilitate computation of the minimum sum to be invested to meet the manager's objective.

Answer:

Shares	**Dividend**	**Growth in Rs.**
A	12%	10/100 = 0.1
B	4%	40/100 = 0.4
Min-income	600	1000

Let x_1 be the amount invested on share A

Let x_2 be the amount invested on share B

Objective function: Min. Z = $x_1 + x_2$

Subject to constraints:

$0.12\ x_1 + 0.04\ x_2 \geq 600$ (Dividend income constraint)

$0.1\ x_1 + 0.4\ x_2 \geq 1000$ (Growth constraint)

And $x_1, x_2 \geq 0$. (Non negativity constraint)

Illustration 3:

A company possesses two manufacturing plants each of which can produce three products X, Y and Z from a common raw material. However, the proportions in which the products are produced are different in each plant and so are the plant's operating costs per hour. Data on production per hour costs are given below, together with current orders in hand for each product.

	Product			Operating cost/hour in Rs.
	X	Y	Z	
Plant A	2	4	3	9
Plant B	4	3	2	10
Orders on hand	50	24	60	

You are required to formulate the problem to find the number of production hours needed to fulfill the orders on hand at minimum cost.

Answer:

Let α be no. of hours of plant A in use
Let β be no. of hours of plant B in use
Objective function: Min Z = $9\alpha + 10\beta$
Subject to constraints:
$2\alpha + 4\beta \geq 50$ (Constraint relating to Product X)
$4\alpha + 3\beta \geq 24$ (Constraint relating to Product Y)
$3\alpha + 2\beta \geq 60$ (Constraint relating to Product Z)
And $\alpha, \beta \geq 0$ (Non negativity constraint)

Illustration 4:

The products P, Q and R are being produced in a plant having profit margin as Rs. 3, Rs. 5 and Rs. 4 respectively. The raw materials A, B and C are of scarce supply and the availability is limited to 8, 15 and 10 units respectively.

Specific consumption is indicated in the table below:

	P	Q	R	Available units
A	2	3	-	8
B	3	2	4	15
C	-	2	5	10
	3/-	5/-	4/-	

Write down the problem mathematically for maximization of profit margin.

Answer:

Let x1 be the no. of units of product P
Let x2 be the no. of units of product Q
Let x3 be the no. of units of product R
Objective function: Max. Z = $3x_1+5x_2 + 4x_3$
Subject to constraints:
$2x_1+3x_2 \leq 8$ (Constraint on availability of Raw Material 'A')
$3x_1+2x_2 + 4x_3 \leq 15$ (Constraint on availability of Raw Material 'B')
$2x_2+5x_3 \leq 10$ (Constraint on availability of Raw Material 'C')
And $x_1, x_2, x_3 \geq 0$ (Non negativity constraint)

Illustration 5:

A Bank is in the process of formulating its loan policy. Involving a maximum of Rs. 600 Million. Table below gives the relevant types of loans. Bad debts are not recoverable and produce no interest receive. To meet competition from other Banks the following policy guidelines have been set. At least 40% of the funds must be allocated to the agricultural and commercial loans. Funds allocated to housing must be at least 50% of all loans given to personal, car, Housing. The overall bad debts on all loans may not exceed 0.06.

Formulate a linear program Model to determine optimal loan allocations.

Type of loan	Interest rate %	Bad debts (Probability)
Personal	17	0.10

Car	14	0.07
Housing	11	0.05
Agricultural	10	0.08
Commercial	13	0.06

Answer:

Let x_1 be the amount allocated for personal loan
Let x_2 be the amount allocated for car loan
Let x_3 be the amount allocated for Housing loan
Let x_4 be the amount allocated for agricultural loan
Let x_5 be the amount allocated for Commercial loan
Objective Function: Max Z
$= 0.17x_1 + 0.14x_2 + 0.11x_3 + 0.1x_4 + 0.13x_5 – (0.10x_1 + 0.07x_2 + 0.05x_3 + 0.08x_4 + 0.06x_5)$
$= (0.17 - 0.10)x_1 + (0.14 - 0.07)x_2 + (0.11 - 0.05)x_3 + (0.10 - 0.08)x_4 + (0.13 – 0.06)x_5$
$= 0.17x_1 + 0.07x_2 + 0.06x_3 + 0.02x_4 + 0.07 x_5$

Subject to constraints

(i) $x_1 + x_2 + x_3 + x_4 + x_5 \leq 600$ Millions (Constraint on total loan amount)

(ii) $x_4 + x_5 \geq 0.4 (x_1 + x_2 + x_3 + x_4 + x_5)$ (Constraint due to policy set for Agricultural and Commercial Loan)

(iii) $x_3 \geq 0.5 (x_1 + x_2 + x_3)$ (Constraint due to policy set for Housing Loan)

(iv) $0.1x_1 + 0.07x_2 + 0.05x_3 + 0.08x_4 + 0.06 x_5 \leq 0.06$ Million (Constraint on limit of overall bad debt)

(v) $x_1, x_2, x_3, x_4, x_5 \geq 0$ (Non negativity constraint)

Illustration 6:

The annual hand-made furniture show and sales occurs next month and the school of vocational studies is planning to make furnitures for sale. There are three wood working classes – I year, II year, III year at the school and they have decided to make three styles of chairs A, B and C. Each chair must receive work in each class and the time in hours for each chair in each class is given.

Chair	I year	II year	III year
A	2	4	3
B	3	3	2
C	2	1	4

In the next month there will be 120 hours available in first year class, 160 hours in the second-year class and 100 hours in the third-year class to produce chairs. The teacher of the wood working class feels that a maximum of 40 chairs can be sold at the show. The teacher has determined that the profit from each type of chair will be A – Rs.40, B – Rs.35 and C – Rs.30.

Formulate a linear programming model to determine how many chairs should be produced to maximize profit.

Answer:

Let x_1 be the chairs produced of A type
x_2 be the chairs produced of B type
x_3 be the chairs produced of C type
Objective function
Maximise $Z = 40x_1 + 35x_2 + 30x_3$

Subject to constraints:

$2x_1 + 3x_2 + 2x_3 \leq 120$ (Constraint on available time of 1st year class)
$4x_1 + 3x_2 + x_3 \leq 160$ (Constraint on available time of 2nd year class)
$3x_1 + 2x_2 + 4x_3 \leq 100$ (Constraint on available time of 3rd year class)
$x_1, x_2, x_3 \geq 0$ (Non negativity constraint)

Illustration 7:

A company produces three products P, Q and R from three raw materials A, B and C. One unit of product P requires 2 units of A and 3 units of B. One unit of product Q requires 2 units of B and 5 units of C and one unit of product R requires 3 units of A, 2 units of B and 4 units of C. The company has 8 units of material A, 10 units of material B and 15 units of material C available to it. Profits per unit of products P, Q and R are Rs. 3, Rs. 5 and Rs.4 respectively.

Formulate the question mathematically to maximize the profit.

Answer:

DATA SUMMARY CHART

Decision variables	Products	Type of raw material			Profits per unit
		A	B	C	(Rs.)
x_1	P	2	3	-	3
x_2	Q	-	2	5	5
x_3	R	3	2	4	4
Units of material available:		8 Maximum	10 maximum	15 maximum	

x_1 = number of units of Product P

x_2 = number of units of Product Q

x_3 = number of units of Product R

The given Q is formulated as the LP model as follows:

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints :

$2x_1 + 3x_3 \leq 8$ (Constraint due to availability of Material A)

$3x_1 + 2x_2 + 2x_3 \leq 10$ (Constraint due to availability of Material B)

$5x_2 + 4x_3 \leq 15$ (Constraint due to availability of Material C)

$x_1, x_2, x_3, \geq 0$ (Non negativity constraint)

Illustration 8:

A city hospital has the following minimal daily requirement for nurses:

Period	Clock time (24 hours day)	Minimal Number of Nurses Required
1	6 a.m. - 10 a.m.	2
2	10 a.m. - 2 p.m.	7
3	2 p.m. - 6 p.m.	75
4	6 p.m. - 10 p.m.	8
5	10 p.m. - 2 a.m.	20
6	2 a.m. - 6 a.m.	6

Nurses report to the hospital at the beginning of each period and work for 8 consecutive hours. The hospital wants to determine the minimal number of nurses to be employed so that there will be sufficient number of nurses available for each period.

Formulate this as a Linear Programming question by setting up appropriate constraints and objective function.

Answer:

$x, + x_3 \geq 15$, $x_3 + x_4 \geq 8$, $x_4 + x_5 \geq 20$, $x_5 + x_6 \geq 6$, and $x_6 + x_1 \geq 2$.

Since, the objective is to minimize the total number of nurses employed in the hospital,

$Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$.

Obviously, we must have $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$.

Illustration 9:

A marketing manager wishes to allocate his annual advertising budget of Rs. 20,000 in two media vehicles A and B. The unit cost of a message in media A is Rs. 1,000 and that of B is Rs. 1,500. Media A is a monthly magazine and not more than one insertion is desired in one issue. At least 5 messages should appear in media B. The expected effective audience for unit messages in the media A is 40,000 and for media B is 55,000.

(i) Develop a mathematical model

Answer:

Step 1. The appropriate mathematical formulation of the given Q…. is as follows:

Maximize (total effective audience) $Z = 40{,}000\ x_1 + 55{,}000\ x_2$

Subject to the constraints

$1{,}000x_1 + 1{,}500x_2 \leq 20{,}000$ (Budget constraint)

$x_1 \leq 12$ (Constraint on annual no. of insertions in Media A)

$x_1 \geq 5$ or $-x2 \leq -5$ (Constraint on annual no. of insertions in Media B)

$x_1, x_2 \geq 0$ (Non negativity constraint)

where

x_1 = annual number of insertions/messages for media A.

x_2 = annual number of insertions/ messages for media B.

Illustration 10:

One unit of product A contributes Rs. 7 and requires 3 units of raw material and 2 hours of labour. One unit of product B contributes Rs. 5 and requires one unit of raw material and one hour of labour. Availability of raw material at present is 48 units and hence there are 40 hours of labour.

i. Formulate it as a linear programming problem.

ii. Write its dual.

Answer:

i. The mathematical formulation of the linear programming problem is

Maximise $Z = 7x_1 + 5x_2$

Subject to $3x_1 + x_2 \leq 48$

$2x_1 + x_2 \leq 40$

$x_1, x_2 \geq 0$

Where x_1 and x_2 denote the number of units of product A and B respectively.

ii. The dual of the above problem is:

Minimize $Z^* = 48y_1 + 40y_2$

$3y_1 + 2y_2 \geq 7$

$y_1 + y_2 \geq 5$

$y_1, y_2 \geq 0$

Where y_1 and y_2 are the dual variables indicating the shadow prices of raw material and labour respectively

Illustration 11:

A Company produces the products P, Q and R from three raw materials A, B and C. One unit of product P requires 2 units of A and 3 units of B. A unit of product Q requires 2 units of B and 5 units of C and one unit of product R requires 3 units of A, 2 unit of B and 4 units of C. The Company has 8 units of material A, 10 units of B and 15 units of C available to it. Profits/unit of products P, Q and R are Rs.3, Rs.5 and Rs.4 respectively.

(a) Formulate the problem mathematically,

(b) Write the Dual problem.

Answer:

Raw Materials	x1	x2	x3	Available units
	P	Q	R	
A	2	-	3	8
B	3	2	2	10
C	-	5	4	15
	3	5	4	

Profits 3/- 5/- 4/-

Let x_1 be the no. of units of P

Let x_2 be the no. of units of Q

Let x_3 be the no. of units of R

Objective function: Max. $Z = 3x_1 + 5x_2 + 4x_3$

Subject to constraints:

$2x_1 + 3x_2 \leq 8$ (Constraint on availability of Raw Material 'A')

$3x_1 + 2x_2 + 2x_3 \leq 10$ (Constraint on availability of Raw Material 'B')

$5x_2 + 4x_3 \leq 15$ (Constraint on availability of Raw Material 'C')

And $x_1, x_2, x_3 \geq 0$. (Non negativity constraint)

Primal

Max. $Z = 3x_1 + 5x_2 + 4x_3$

Subject to

$2x_1 + 3x2 \leq 8$

$3x_1 + 2x_2 + 2x_3 \leq 10$

$5x_2 + 4x_3 \leq 15$

And $x_1, x_2, x_3 \geq 0$

Dual

Min. $Z = 8y_1 + 10y_2 + 15y_3$

Subject to

$2y_1 + 3y_2 \geq 3$

$3y_1 + 2y_2 + 5y_3 \geq 5$

$2y_2 + 4y_3 \geq 4$

And $y_1, y_2, y_3 \geq 0$

$2x_1 + 3x_2 + S_1 = 8$

$3x_1 + 2x_2 + 2x_3 + S_2 = 10$

$5x_2 + 4x_3 + S_3 = 15$

Max $Z = 3x_1 + 5x_2 + 4x_3 + 0.S_1 + 0.S_2 + 0.S_3$

$\therefore x_1 = 23/20 \qquad x_2 = 19/10 \qquad x_3 = 11/8$

$Z = 18.45$

Illustration 12:

Four Products A, B, C and D have Rs. 5, Rs. 7, Rs. 3 and Rs. 9 profitability respectively. First type of material (limited supply of 800 kgs.) is required by A, B, C and D at 4 kgs., 3 kgs, 8 kgs, and 2 kgs. respectively per unit.

Second type of material has a limited supply of 300 kgs. and is for A, B, C and D at 1 kg, 2 kgs, 0 kgs, and 1 kg per unit. Supply of the other type of materials consumed is not limited. Machine hrs. available are 500 hours and the requirements are 8,5,0 and 4 hours for A, B, C and D each per unit.

Labour hours are limited to 900 hours and requirements are 3,2,1 and 5 hours for A, B, C and D respectively.

How should the firm approach so as to maximize its profitability? Formulate this as a linear programming problem. You are not required to solve the LPP.

Answer:

Let x_1 be the no. of units of product A
Let x_2 be the no. of units of product B
Let x_3 be the no. of units of product C
Let x_4 be the no. of units of product D
Objective function Maximize $Z = 5x_1+7x_2+3x_3+9x_4$

	A	B	C	D	Supply in Kgs.
I type material	4	3	8	2	800
II type material	1	2	0	1	300
Machine	8	5	0	4	500
Labour	3	2	1	5	900
Profit	5	7	3	9	

Subject to constraints
$4x_1+3x_2+ 8x_3+2x_4 \leq 800$ (Constraint on availability of Material type I)
$x_1+2x_2+ 0.x_3+x_4 \leq 300$ (Constraint on availability of Material type II)
$8x_1+5x_2+ 0.x_3+4x_4 \leq 500$ (Constraint on Machine Hours available)
$3x_1+2x_2+ x_3+5x_4 \leq 900$ (Constraint on Labour Hours available)
and $x_1, x_2, x_3, x_4 \geq 0$. (Non negativity constraint)

Illustration 13:

Mutual Fund has cash resources of Rs. 200 million for investment in a diversified portfolio. Table below shows the opportunities available, their estimated annual yields, risk factor and term period details. Formulate a Linear Program Model to find the optimal portfolio that will maximize return, considering the following policy guidelines:

- All the funds available may be invested
- Weighted average period of at least five years as planning horizon.
- Weighted average risk factor not to exceed 0.20.
- Investment in real estate and speculative stocks to be not more than 25% of the monies invested in total.

Investment type	Annual yield (percentage)	Risk factor	Term period (years)
Bank deposit	9.5	0.02	6
Treasury notes	8.5	0.01	4
Corporate deposit	12.0	0.08	3
Blue-chip stock	15.0	0.25	5
Speculative stocks	32.5	0.45	3
Real estate	35.0	0.40	10

Answer:

Let x_1, x_2, x_3, x_4, x_5 and x_6 represent the six different investment alternatives, i.e., x_1 is bank deposit, x_2 is treasury note. X_3 corporate deposit, X_4 blue chip stock, X_5 speculative stock and x_6 real estate. The objective is to maximize the annual yield of the investors (in number of units) given by the linear expression.

Maximize $Z = 9.5x_1 + 8.5x_2 + 12.0x_3 + 15.0x_4 + 32.5x_5 + 35.0x_6$
Subject to the Constraints:
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 1$ (Investment decision)
$0.02x_1 + 0.01x_2 + 0.08x_3 + 0.25x_4 + 0.45x_5 + 0.40x_6 \leq 0.20$ (Constraint on weighted average risk of the portfolio)
$6x_1 + 4x_2 + 3x_3 + 5x_4 +3x_5 + 10x_6 \geq 5$ (Constraint on weighted average length of period of investment)
$x_5 + x_6 \leq 0.25$ (Constraint on investment in real estate and speculated stock)
$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ (non-negativity condition)

Illustration 14:

What is the slope of the objective function Max $Z = 15X + 45Y$?

Answer:

The slope form is $Y = mX + b$ where m = slope

Rearranging,

$45Y = -15X + Z$

$$y = \frac{15x}{45} + \frac{z}{45}$$

Slope is –15/45 or – 1/3.

Illustration 15:

An electronic goods manufacture has distributors who will accept shipments of either transistor radios or electronic calculators to stock for Christmas inventory Whereas the radios contribute Rs.10 per unit and the calculator Rs.15 per unit to profits, both products use some of the same components. Each radio requires each of diodes and resistors, while each calculator requires 10 diodes and 2 resistors. The radio take 12.0 minutes and the calculators take 9.6 minutes of time on the company's electronic testing machine, and the production manager estimates that 160 hours of test time is a available. The firm has 8,000 diodes and 3,000 resistors in inventory. What product of mix of products should be selected to obtain the highest profit?

Answer:

The decision variables are radios, R, and calculators, C, and we must determine how many of each should be produced to maximize profit, Z.

(1) Objective function

Max $Z = 10R + 15C$

Constraints

Diodes (8,000 available): Radios require 4 each, and calculators require 10 each.

$\therefore 4R + 10C \leq 8,000$

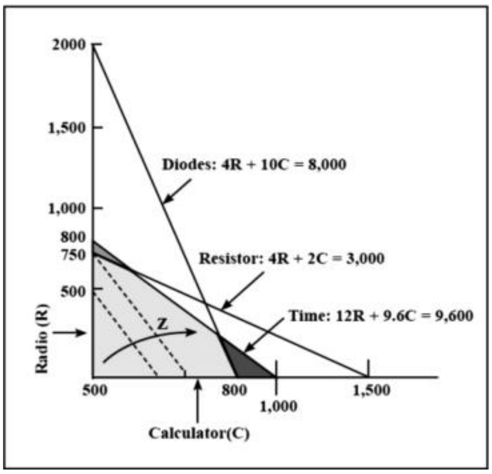
Resistors (3,000 available): Radios require 4 each, and calculators require 2 each.

$\therefore 4R + 2C \leq 3,000$

Testing (9,600 minutes available): Radios require 12.0 minutes, and calculators require 9.6 minutes.

$12R + 9.6C \leq 9,600$

(2) Graph 'bf variables and constraints



Plotting each of the constraints inequality as an equality, we have:

For Diodes: $4R + 10C = 8000$

If $R = 0$, then $C = 800$

If $C = 0$, then $R = 2,000$

For Resistors: $4R + 2C = 3,000$

If $R = 0$, then $C = 1500$

If $C = 0$, then $R = 750$

For Testing: $12R + 9.6C = 9,600$

If $R = 0$, then $C = 1,000$

If $C = 0$, then $R = 800$

Note: The resulting graph establishes a feasible region bounded by the time, diode, and resistor constraints that $R \geq 0$ and $C \geq 0$.

(3) Slope of the objective function.

We can express our objective function in slope intercept from, where the Y axis corresponds to R and the X axis to C.

$Z = 10R + 15C$

Or, 10R = -15C + Z

$\therefore R = \frac{15}{10}C + \frac{z}{10} = \frac{3}{2}C + \frac{z}{10}$

∴ Slope = – 3/2, which means that for every 3-unit decrease in Y there is a 2 increase in X. This slope is plotted as a dotted line in the graph by marking off 3 units (negative) in R for each 2 units (positive) in C.

(4) Move objective function to optimize. The slope of the objective function hi to objective line) is moved away from the origin until constrained. In this case the binding constraints are the diode inventory supply and testing machine time availability.

(5) Read solution values. The arrows point to the approximate R and C coordinates o the constraining intersection.

Number of radios = 240

Number of calculators = 700

Note: That the simultaneous solution of the two binding constraint equations would lend more accuracy to the answer:

(4R + 10C = 8,000) × (-3) = – 12R – 30C = – 24,000

$\underline{12R + 9.6C \quad = \quad 9{,}600}$

-20.4C = – 14,400

C = 705 calculators

Substituting to solve for R:

4R + 10(705) = 8,000

∴ R = 8,000 – 7,050/ 4 = 237 radios

Comment: We had two decision variables (that is, products) to choose from and established a profit function, Z, and constraints and optimized the function by moving it away from the origin. The graph of this example showed that the resistor supply was not constraining, so only two constraints (diodes and test time) were binding. Similarly, there were two decision variables in the solution, that is, we ended up producing both radios and calculators. The number of variables in solution will always equal the number of explicit constraints that are binding.

The graphic linear programming solution gives an indication of the sensitivity of the solution to changes in the constraints. If for example, additional diodes could be purchased from an outside supplier with no increase in cost, profit would be maximized by extending the iso – objective line to the next corner and producing 1,000 calculators and no radios. In this case we would have one explicit constraint (time) binding and only one decision variables (calculators) in the final solution.

Illustration 16:

The simplex calculator company makes a profit of Rs.5 on each model X and Rs.20 on each model Y. Each calculator requires the following time (in minutes) on the cleaning and testing machines.

	X Requirements	Y Requirements	Time Available
Cleaning Testing	2 6	4 3	10 12

(a) State the objective function and constraints.

(b) Arrange the equations in a simplex format.

Answer:

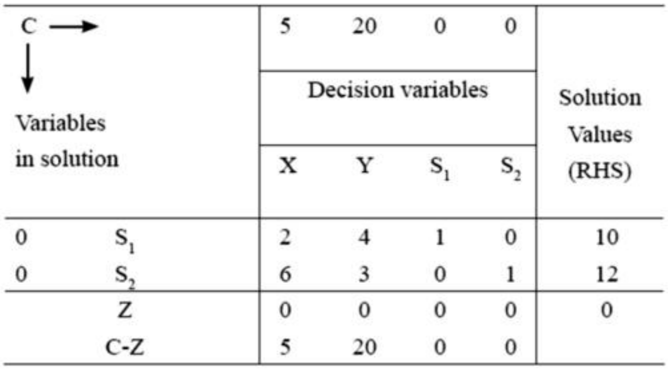
(a) Objective function Max Z = 5X + 20Y

Constraints:

Cleaning $2X + 4Y \leq 10$

Testing $6X + 3Y \leq 12$

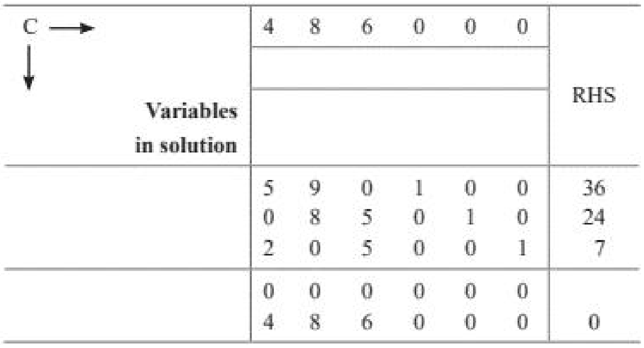
(b)



C → ↓ Variables in solution		5	20	0	0	Solution Values (RHS)
		Decision variables				
		X	Y	S_1	S_2	
0	S_1	2	4	1	0	10
0	S_2	6	3	0	1	12
	Z	0	0	0	0	0
	C-Z	5	20	0	0	

Illustration 17:

The initial matrix of a maximization linear programming problem is as shown where the decision variables are designated A, B, etc.



C → ↓ Variables in solution	4	8	6	0	0	0	RHS
	5	9	0	1	0	0	36
	0	8	5	0	1	0	24
	2	0	5	0	0	1	7
	0	0	0	0	0	0	
	4	8	6	0	0	0	0

(a) State the original constraint equations.

(b) How many decision variables are there?

(c) State the objective function.

Answer:

(a) 5A + 9B ≤ 36, 8B + 5C ≤ 24, and 2A + 5C ≤ 7

(b) Three

(c) Max Z = 4A + 8B + 6C

4.5 Transportation Problem

Questions For Classroom Discussion

Question 1

Find basic feasible (Initial Solution) solution to the transportation problem?

Answer 1: Solve through Northwest Corner Rule Method

Answer 2: Solve through Lowest Cost Entry Method

Answer 3: Solve through Vogel's Approximation Method (Penalty Method)

Question 2

Find optimal solution to the transportation Problem?

Answer

Step 1: Find basic feasible solution through Vogel's Approximation method.

Step 2: Find optimal solution through Modified Distribution Method. (MODI's Method) or (U-V) Method.

Question 3

Needline Exporters is a Tirurangadi-based textile firm exporting textile items to Dubai, New York, and Manama City from its four factories located in Madhura, Ahmedabad, Sivakasi, and Chennai. The firm has received the following requirements:

He got a requirement of 7000, 9000 and 18000 from Dubai, New York and Manama city respectively.

The Quantity available at Madhura, Ahmedabad, Sivakasi and Chennai are 5000, 7000, 8000, and 14000 respectively.

The per unit cost of transportation from different origins to destinations are given in the table below.

Find basic feasible solution to the transportation problem through North West Corner Rule Method .

Origin/Destination	Dubai	New York	Manama	**Supply**
Madhura	2	7	4	5000
Ahmedabad	3	3	1	7000
Sivakasi	5	4	7	8000
Chennai	1	6	2	14000
Demand	7000	9000	18000	**34000**

Question 4

Solve the following transportation problem, through North West Corner Rule Method

Factory/Warehouse	W1	W2	W3	W4	W5	**Supply**
F1	3	4	6	8	9	20
F2	2	10	1	5	8	30
F3	7	11	20	40	3	15
F4	2	1	9	14	16	13
Demand	40	6	8	18	6	**78**

Question 5 **Home Work Question (Previous Year Question)**

Solve the following transportation problem, through Northwest corner rule

Plant\ Warehouse	W1	W2	W3	W4	Supply
P1	190	300	500	100	70
P2	700	300	400	600	90
P3	400	100	600	200	180
Demand	50	80	70	140	

Lowest cost Entry Method and Vogels Approximation Method

The above questions are solved using the following methods:

1. Least Cost Entry Method
2. Vogel's Approximation Method (Penalty Method)

Modified Distribution Method- (MODI's Method) or (U-V) Method

Find optimal solution to the transportation from the basic feasible solution.

	W1	W2	W3	W4	W5	Demand
F1	$3^{[20]}$	4	6	8	9	20
F2	$4^{[4]}$	10	$1^{[8]}$	$5^{[18]}$	8	30
F3	$7^{[9]}$	11	20	40	$3^{[6]}$	15
F4	$2^{[7]}$	$1^{[6]}$	9	14	16	13
Supply	40	6	8	18	6	**78**

Question 6 Practice Question

Find optimal solution to the Transportation Problem from the basic feasible solution.

	W1	W2	W3	Supply
F1	$2^{[5]}$	7	4	5
F2	3	$3^{[2]}$	$1^{[6]}$	8
F3	5	$4^{[7]}$	7	7
F4	$1^{[2]}$	6	$2^{[12]}$	14
Demand	7	9	18	**34**

Unbalanced Transportation Problem

Unbalanced Transportation Problems

Transportation problems are considered unbalanced when:

1. Supply is greater than demand
2. Demand is greater than Supply

Question 7

Find basic feasible solution to the transportation problem? Which Factories items are unsold?

	W1	W2	W3	W4	Supply
F1	11	20	7	8	50
F2	21	16	10	12	40
F3	8	12	18	9	70
Demand	30	25	35	40	130\160

Maximization of the Transportation Problem

Question 8

Solve the following maximization of Transportation Problem?

	W1	W2	W3	W4	Supply
F1	15	51	42	33	23
F2	80	42	26	81	44
F3	90	40	66	60	33
Demand	23	31	16	30	**100**

Mathematical Formulation of LPP

Formulate LPP to the minimization problem

	W1	W2	W3	Supply
F1	3	5	8	25

F2	8	9	10	35
F3	4	7	9	40
Demand	20	30	50	**100**

Mathematical Formulation of LPP (Unbalanced)
Formulate LPP to the minimization problem

	W1	W2	W3	Supply
F1	3	5	8	25
F2	8	9	10	35
F3	4	7	9	40
Demand	20	30	60	110\100

Question 9 **Homework – Previous Year Question (2007, June 3(a))**

The following table shows all the necessary information on the availability of supply to each factory of Agro Industries Ltd, the requirement of each destination and the unit of transport cost from each factory to each destination

	W1	W2	W3	Supply
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
Demand	75	20	50	145\105

Since there is not enough supply, some of the demand at three destination may not be satisfied. For the unsatisfied demands, penalty costs be Re 1, 2 and 3 respectively.

Find the optimal allocation that minimize the transportation and penalty costs by using the Vogel's Approximation Method (VAM) (10 Marks)
Optimal Answer is 555

Illustration From Study Material

Illustration 18:

The cost conscious company requires for the next month 300, 260 and 180 tonnes of stone chips for its three constructions C1, C2 and C3 respectively. Stone chips are produced by the company at three mineral fields taken on short lease by the company. All the available boulders must be crushed into chips. Any excess chips over the demands at sites C1, C2 and C3 will be sold ex-fields.

The fields are M1, M2 and M3 which will yield 250, 320 and 280 tons of stone chips respectively.

Transportation costs from mineral fields to construction sites vary according to distances, which are given below in monetary unit (MU).

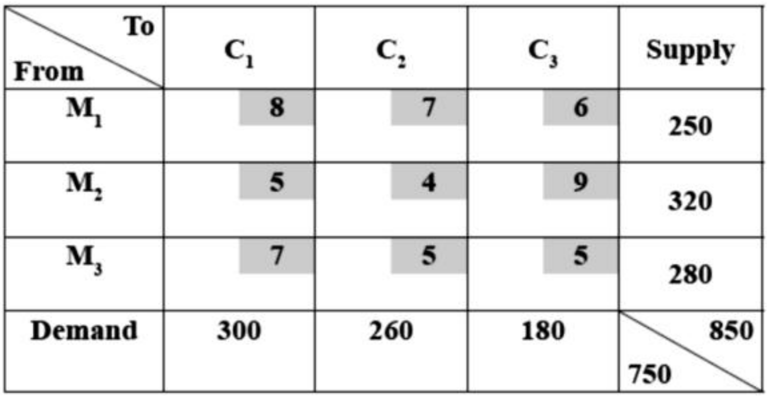
	To	C1	C2	C3
From	M1	8	7	6
	M2	5	4	9
	M3	7	5	5

(i) Determine the optimal economic transportation plan for the company and the overall transportation cost in MU.

(ii) What are the quantities to be sold from M1, M2 and M3 respectively?

Answer:

(i) Table: 1 Cost Matrix

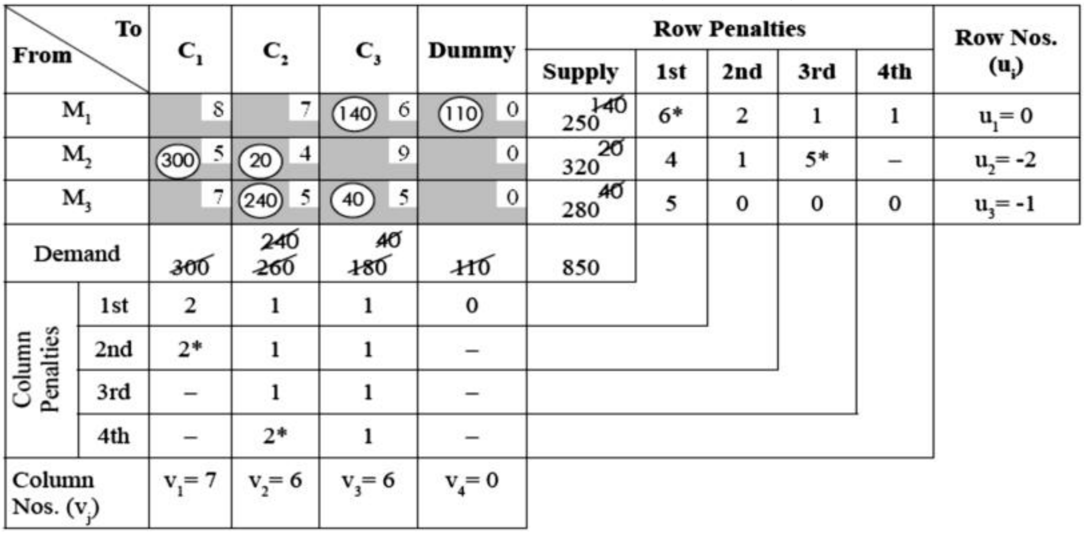


To / From	C_1	C_2	C_3	Supply
M_1	8	7	6	250
M_2	5	4	9	320
M_3	7	5	5	280
Demand	300	260	180	850 750

From the given data we have Total Supply = 850 tonnes and total and total Demand = 740 tonnes i.e., Supply ≠ Demand.

So this is an unbalanced problem of transpartation. To make it balanced we introduce a “Dummy” construction site of demand 850 – 740 = 110 tonnes and having zero cost elements for all the cells of the matrix corresponding to it.

Table: 2 Basic Feasible Solution by VAM (Optimal)



To / From		C_1	C_2	C_3	Dummy	Row Penalties					Row Nos. (u_i)
						Supply	1st	2nd	3rd	4th	
M_1		8	7	140 6	110 0	250 ~~140~~	6*	2	1	1	$u_1 = 0$
M_2		300 5	20 4	9	0	320 ~~20~~	4	1	5*	–	$u_2 = -2$
M_3		7	240 5	40 5	0	280 ~~40~~	5	0	0	0	$u_3 = -1$
Demand		~~300~~	~~240~~ ~~260~~	~~40~~ ~~180~~	~~110~~	850					
Column Penalties	1st	2	1	1	0						
	2nd	2*	1	1	–						
	3rd	–	1	1	–						
	4th	–	2*	1	–						
Column Nos. (v_j)		$v_1 = 7$	$v_2 = 6$	$v_3 = 6$	$v_4 = 0$						

Row Penalty = 2nd lowest cost figure of a row – Lowest cost figure of that row.

For the **1st Set of Row Penalties** –

(a) For 1st row, 2nd lowest cost = 6 and lowest cost = 0
∴ Penalty = 6 – 0 = 6

(b) For 2nd Row, 2nd lowest cost = 4 and Lowest cost = 0
∴ Penalty = 4 – 0 = 4

(c) For 3rd Row, 2nd lowest cost = 5 and Lowest cost = 0,
∴ Penalty = 5 – 0 = 5

Similarly, **Column Penalty** = 2nd lowest cost figure of a column – Lowest cost figure of that column

For the **1st Set** of **Column Penalties** –

(a) For 1st column, 2nd lowest cost = 7 and Lowest cost = 5, ∴ Penalty = 7 – 5 = 2

(b) For 2nd column, 2nd lowest cost = 5 and Lowest cost = 4, ∴ Penalty = 5 – 4 = 1

(c) For 3rd column, 2nd lowest cost = 6 and Lowest cost = 5, ∴ Penalty = 6 – 5 = 1

Of all these Row and Column penalties of 1st set, 6 is highest and it corresponds to 1st Row.

Hence allocation should be done at that cell of 1st Row where cost is least. This corresponds to the cell (M1 – Dummy). So maximum possible unit of 110 is allocated in this cell by maintaining parity of supply and demand.

With this allocation the total demand of 'Dummy' site is exhausted. But the supply of the corresponding Mineral Field (M1) is not fully exhausted. Remaining supply capacity of M1 i.e. 250 – 110 = 140 tonnes is shown as balance in the supply cell of M1. As the demand of 'Dummy' is fullfilled, the entire column for this has been shaded indicating the same. Figures of this column will no longer participate in any of the subsequent calculations of Penalty (for Rows as well as columns)

The same procedure of calculating penalty for Rows and Columns and subsequently allocating maximum possible quantity in the least cost cell corresponding to highest penalty is repeated until all the allocations are made maintaining parity of Supply and Demand.

The solution thus obtained is the **Basic Feasible Solution**. It is given as follows.

Table: Showing Optimum Allocation

Cell	**Allocation**	**Cost of Transportation (Rs.)**
M1 - C3	140 tonnes	140 × 6 = 840
M1 - Dummy	110 tonnes	110 × 0 = 0
M2 - C1	300 tonnes	300 × 5 = 1500
M2 - C2	20 tonnes	20 × 4 = 80
M3 - C2	240 tonnes	240 × 5 = 1200
M3 - C3	40 tonnes	40 × 5 = 200
Total	850 tonnes	Rs. 3820

Here, m = No. of rows of the matrix = 3

n = No. of columns of the matrix = 4

∴ m + n – 1 = 3 + 4 – 1 = 6

Also, no. of allocated cells = 6

As, no. of allocated cells = 6 = m + n – 1, the solution is a **non degenerate** one.

Now the solution is tested for **OPTIMALITY**.

For this, Row Nos. (u_i) and column nos. (v_j) are calculated by using the equation $C_{ij} = u_i + v_j$ for all the **allocated cells**, where C_{ij} = Cost figure of the cell i-j.

Allocated Cell	C_{ij}	$C_{ij} = u_i + v_j$			
M_1 - C_3	$C_{13} = 6$	$C_{13} = u_1 + v_3$	or, $6 = u_1 + v_3$	or, $6 = 0 + v_3$ (Assume $u_1 = 0$) or, $v_3 = 6$	**(1)**
M_1 - Dummy	$C_{14} = 0$	$C_{14} = u_1 + v_4$	or, $0 = u_1 + v_4$	or, $0 = 0 + v_4$ or, $v_4 = 0$	**(2)**
M_2 - C_1	$C_{21} = 5$	$C_{21} = u_2 + v_1$	or, $5 = u_2 + v_1$	or, $5 = -2 + v_1$ or, $v_1 = 7$	**(6)**
M_2 - C_2	$C_{22} = 4$	$C_{22} = u_2 + v_2$	or, $4 = u_2 + v_2$	or, $4 = u_2 + 6$ or, $u_2 = -2$	**(5)**
M_3 - C_2	$C_{32} = 5$	$C_{32} = u_3 + v_2$	or, $5 = u_3 + v_2$	or, $5 = -1 + v_2$ or, $v_2 = 6$	**(4)**
M_3 - C_3	$C_{33} = 5$	$C_{33} = u_3 + v_3$	or, $5 = u_3 + v_3$	or, $5 = u_3 + 6$ or, $u_3 = -1$	**(3)**

Hence no. of equations = 6 and no. of unknowns = 7. So to start with a solution, it is assumed u1 = 0. Thereafter all the other row nos. and column nos. are calculated. The sequence of usage of the above equations is indicated as (1), (2), (3), ... (6).

Next opportunity cost (Δ_{ij}) for all the unallocated cells are calculated using $\Delta_{ij} = C_{ij} - (u_i + v_j)$

Unallocated Cell	C_{ij}	**Opportunity Cost** $[\Delta_{ij} = C_{ij} - (u_i + v_j)]$
M_1 - C_1	$C_{11} = 8$	$\Delta_{11} = C_{11} - (u_1 + v_1) = 8 - (0 + 7) = 1$
M_1 - C_2	$C_{12} = 7$	$\Delta_{12} = C_{12} - (u_1 + v_2) = 7 - (0 + 6) = 1$
M_2 - C_3	$C_{23} = 9$	$\Delta_{23} = C_{23} - (u_2 + v_3) = 9 - (-2 + 6) = 5$
M_2 - Dummy	$C_{24} = 0$	$\Delta_{24} = C_{24} - (u_2 + v_4) = 0 - (-2 + 0) = 2$
M_3 - C_1	$C_{31} = 7$	$\Delta_{31} = C_{31} - (u_3 + v_4) = 7 - (-1 + 7) = 1$
M_3 - Dummy	$C_{34} = 0$	$\Delta_{34} = C_{34} - (u_3 + v_4) = 0 - (-1 + 0) = 1$

As all the opportunity cost values are nonnegative, the solution is optimal.

(i) So the optimal transportation plan is as shown in Table-3 and minimum cost of transportation isRs. 3820/-

(ii) Quantities to be produced by M1, M2 and M3 are respectively 250,320 and 280 tonne of which 110 tonnes worth of stone chips produced by M1 will remain unused by the construction sites. So this quantity can be sold ex-field.

Illustration 19:

Ladies fashion shop wishes to purchase the following quantity of summer dresses:

Dress size	I	II	III	IV
Quantity	100	200	450	150

Three manufacturers are willing to supply dresses.

The quantities given below are the maximum that they are able to supply of any given combination of orders for dresses:

Manufacturers	A	B	C
Total quantity	150	450	250

The shop expects the profit per dress to vary with the manufacturer as given below:

Size

	I	II	III	IV
A	Rs.2.5	Rs.4.0	Rs.5.0	Rs.2.0
B	Rs.3.0	Rs.3.5	Rs.5.5	Rs.1.5
C	Rs.2.0	Rs.4.5	Rs.4.5	Rs.2.5

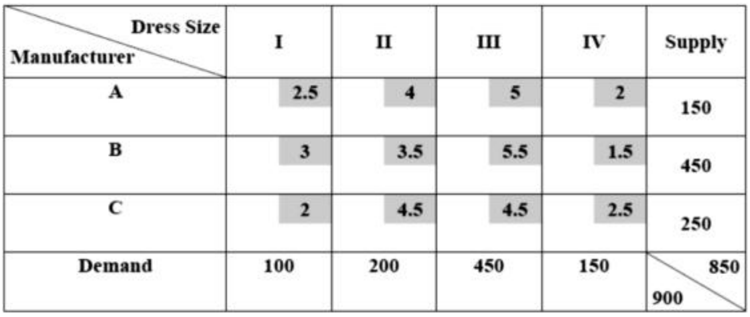
Required:

(a) Use the transportation technique to solve the problem of how the orders should be placed with the manufacturers by the fashion shop is order to maximise profit.

(b) Explain how you know there is no further improvement possible.

Answer:

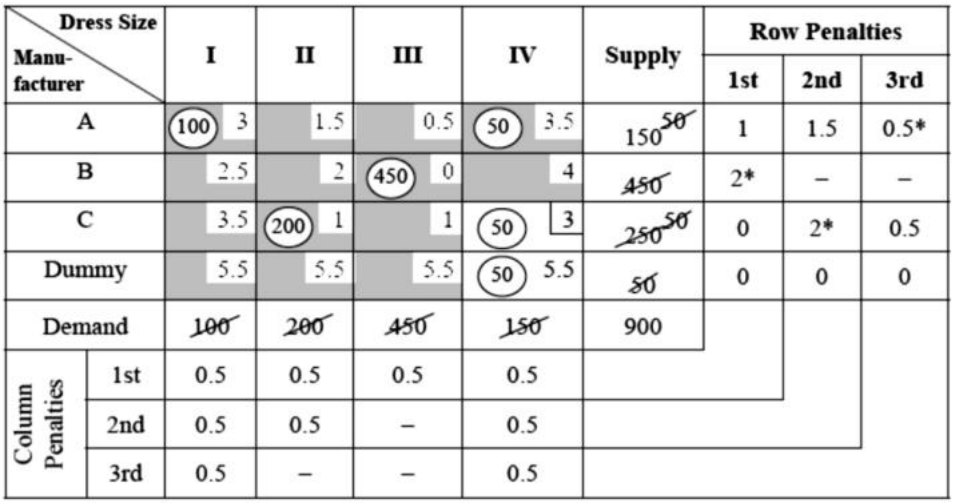
Table: 1 Profit Matrix



Dress Size / Manufacturer	I	II	III	IV	Supply
A	2.5	4	5	2	150
B	3	3.5	5.5	1.5	450
C	2	4.5	4.5	2.5	250
Demand	100	200	450	150	850 900

Maximum possible supply capability of manufacturer = 850 units
Total Demand = 900 units
As Supply ≠ demand, the problem is an unbalanced one. To make it balanced, a 'Dummy' manufacturer of supply capacity = 900 – 850 = 50 units. is introduced. The profit figures for it are all zeros.
Also it is a problem of maximisation, to convert it to a problem of minimisation, a Relative Loss matrix is formed by subtracting all the profit figures given in the above matrix as well as those of Dummy from the highest profit (5.5) figure of the given matrix.

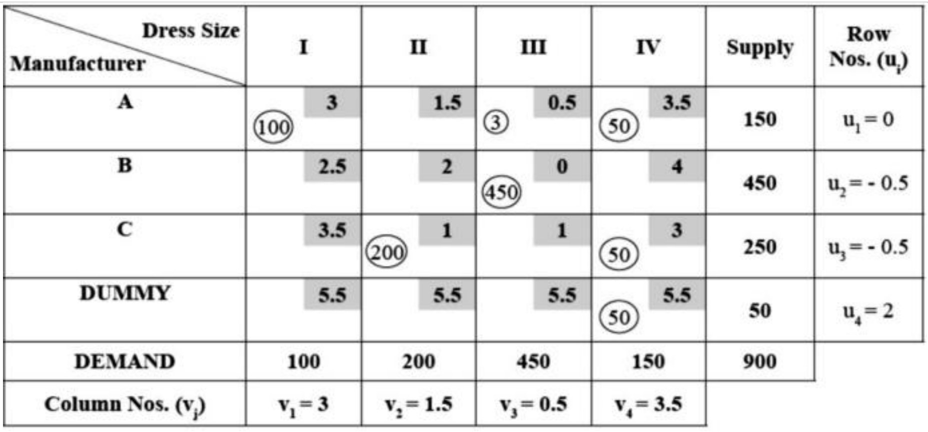
Table: 2 Relative Loss Matrix with Basic Feasible Solution



Dress Size / Manu-facturer		I	II	III	IV	Supply	Row Penalties 1st	2nd	3rd
A		(100) 3	1.5	0.5	(50) 3.5	150 ~~50~~	1	1.5	0.5*
B		2.5	2	(450) 0	4	~~450~~	2*	–	–
C		3.5	(200) 1	1	(50) 3	~~250~~ ~~50~~	0	2*	0.5
Dummy		5.5	5.5	5.5	(50) 5.5	~~50~~	0	0	0
Demand		~~100~~	~~200~~	~~450~~	~~150~~	900			
Column Penalties	1st	0.5	0.5	0.5	0.5				
	2nd	0.5	0.5	–	0.5				
	3rd	0.5	–	–	0.5				

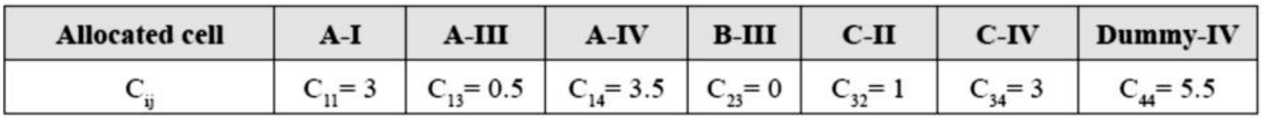
Here, m = No. of rows of the matrix = 4 and n = No. of columns of the matrix = 4
∴ m + n – 1 = 4 + 4 – 1 = 7
Also no. of allocated cells = 6 ≠ (m+ n – 1)
So the solution is a degenerate one. To resolve this, we make use of an artificial quantity 'e' and allocate this quantity at the unallocated cell which is having least cost among all the unallocated cells. It can be mentioned that the quantity 'e' is very small and for all practical purposes its value can be taken as zero.
Least cost unallocated cell is (A-III) where allocation of 'e' has to be made.

Table: 3 Showing Basic Feasible Solution (Optimal)



Dress Size / Manufacturer	I	II	III	IV	Supply	Row Nos. (u_i)
A	3 (100)	1.5	0.5 ③	3.5 ㊿	150	$u_1 = 0$
B	2.5	2	0 (450)	4	450	$u_2 = -0.5$
C	3.5	1 (200)	1	3 ㊿	250	$u_3 = -0.5$
DUMMY	5.5	5.5	5.5	5.5 ㊿	50	$u_4 = 2$
DEMAND	100	200	450	150	900	
Column Nos. (v_j)	$v_1 = 3$	$v_2 = 1.5$	$v_3 = 0.5$	$v_4 = 3.5$		

To test optimality of the Basic Feasible Solution, Row Nos. (ui) and Column Nos. (vj) are calculated using the equation Cij = ui + vj for the allocated cells, where Cij = Relative Loss figure of the cell i - j.



Allocated cell	A-I	A-III	A-IV	B-III	C-II	C-IV	Dummy-IV
C_{ij}	$C_{11} = 3$	$C_{13} = 0.5$	$C_{14} = 3.5$	$C_{23} = 0$	$C_{32} = 1$	$C_{34} = 3$	$C_{44} = 5.5$

$C_{11} = u_1 + v_1$ or, $3 = 0 + v_1$ [$u_1 = 0$, Assumed] or, v1 = 3

$C_{13} = u_1 + v_3$ or, $0.5 = 0 + v_3$ or, $v_3 = 0.5$; $C_{14} = u_1 + v_4$ or, $3.5 = 0 + v_4$ or, $v_4 = 3.5$

$C_{23} = u_2 + v_3$ or, $0 = u_2 + 0.5$ or, $u_2 = -0.5$; $C_{34} = u_3 + v_4$ or, $3 = u_3 + 3.5$ or, $u_3 = -0.5$

$C_{32} = u_3 + v_2$ or, $1 = -0.5 + v_2$ or, $v_2 = 1.5$; $C_{44} = u_4 + v_4$ or, $5.5 = u_4 + 3.5$ or, $u_4 = 2$

Opportunity Loss figures (Δif) for all the unallocated cells are calculated using the equation $\Delta_{ij} = C_{ij} - (u_i + v_j)$

Unallocated Cell	**Opportunity Loss (Δij)**
A - II	$\Delta_{12} = C_{12} - (u_1 + v_2) = 1.5 - (0 + 1.5) = 0$
B - I	$\Delta_{21} = C_{21} - (u_2 + v_1) = 2.5 - (-0.5 + 3) = 0$
B - II	$\Delta_{22} = C_{22} - (u_2 + v_2) = 2 - (-0.5 + 1.5) = 1$
B - IV	$\Delta_{24} = C_{24} - (u_2 + v_4) = 4 - (-0.3 + 3.5) = 1$
C - I	$\Delta_{31} = C_{31} - (u_3 + v_1) = 3.5 - (-0.5 + 3) = 1$
C - III	$\Delta_{33} = C_{33} - (u_3 + v_3) = 1 - (-0.5 + 0.5) = 1$
Dummy - I	$\Delta_{41} = C_{41} - (u_4 + v_1) = 5.5 - (2 + 3) = 0.5$
Dummy - II	$\Delta_{42} = C_{42} - (u_4 + v_2) = 5.5 - (2 + 1.5) = 2$
Dummy - III	$\Delta_{43} = C_{43} - (u_4 + v_3) = 5.5 - (2 + 1.5) = 3$

As all the opportunity loss values are non negative, the solution is optimal.

Table Showing Optimum allocation of orders quantities

From Manufacturer	**Dress Size**	**Allocated Quantity**	**Profit/unit (Rs.)**	**Total (Rs.)**
(i)	(ii)	(iii)	(iv)	(v) = (iii) × (iv)
A	I	100 units	2.5	250
	IV	50 units	2	100
B	III	450 units	5.5	2475
C	II	200 units	4.5	900
	IV	50 units	2.5	125
Dummy	IV	50 units	0	0
Total	-	**900 units**	-	**Rs. 3850**

Maximum Profit = Rs. 3850/-

Illustration 20:

The products of three plants F1, F2 and F3 are to be transported to 5 warehouses W1, W2, W3, W4 and W5. The capacities of plants, demand of warehouses and the cost of transportation from one plant to various warehouses are indicated in the following table:

	W1	W2	W3	W4	W5	Plant Capacity
F1	74	56	54	62	68	400
F2	58	64	62	58	54	500
F3	66	70	52	60	60	600
Warehouse Demand	200	280	240	360	320	1500/1400

(a) Find out a distribution plan of products from plants to the warehouses at a minimum cost. What is the minimum cost?

(b) Is there any surplus capacity of the plants? If so, in which plant should we associate that surplus capacity?

(c) Is there any alternate solution for the optimum solution achieved in

Answer:

(a) From the given data total plant capacity (1500 units) is more than the total demand of warehouses (1400 units). So the problem is unbalanced. To make it balanced, a 'Dummy' warehouse of demand 1500 – 1400 = 100 units is introduced. Cost figures corresponding to various cells of this 'Dummy' are zeros.

Table: 1 Basic Feasible Solution

Warehouse / Plant		W_1	W_2	W_3	W_4	W_5	Dummy	Plant Capacity	Raw Penalities					
									1	2	3	4	5	6
F_1		74	56 280	54	62 120	68	0	~~120~~ ~~400~~	54	2	6	6	6	6
F_2		58 200	64	62	58	54 200	0 100	~~400~~ ~~500~~ ~~200~~	* 54	4	4	4	4	-
F_3		66	70	52 240	60 240	60 120	0	~~600~~ ~~360~~ ~~240~~	52	* 8	0	0	0	0
Warehouse Demand		~~200~~	~~280~~	~~240~~	~~360~~	~~120~~ ~~320~~	~~100~~	1500						
Column Penalities	1	8	8	2	2	6	0							
	2	8	8	2	2	6	-							
	3	8	8*	-	2	6	-							
	4	8*	-	-	2	6	-							
	5	-	-	-	2	6*	-							
	6	-	-	-	2	8*	-							

Here, m = No. of rows = 3

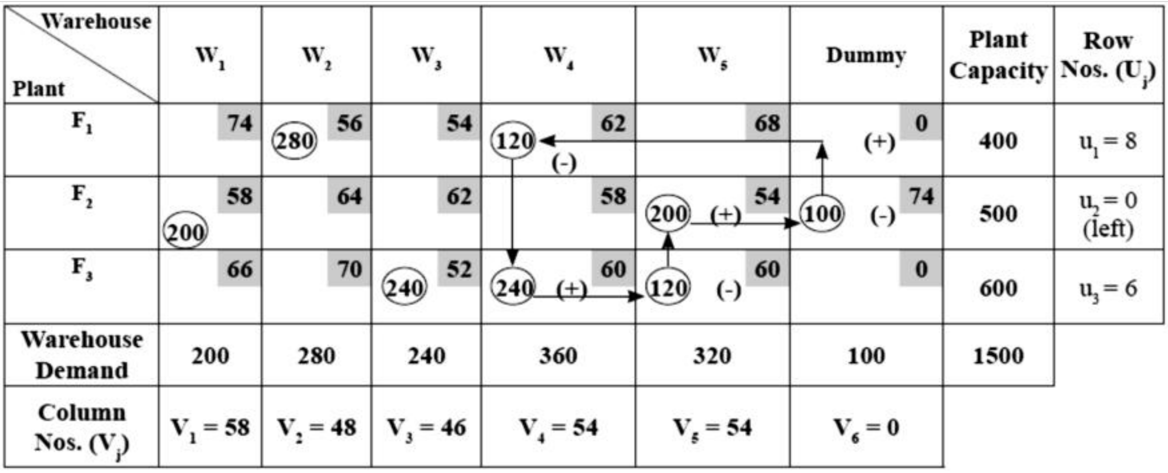
n = No. of columns = 6

m + n - 1 = 3 + 6 - 1 = 8

Also no. of allocated cells = 8 = m + n - 1.

So the solution is nondegenerate.

Table: 2 Showing Basic Feasible Solution (Non Optimal)



Warehouse / Plant	W1	W2	W3	W4	W5	Dummy	Plant Capacity	Row Nos. (U_i)
F1	74	56 280	54	62 120 (-)	68	0 (+)	400	$u_1 = 8$
F2	58 200	64	62	58	54 200 (+)	74 100 (-)	500	$u_2 = 0$ (left)
F3	66	70	52 240	60 240 (+)	60 120 (-)	0	600	$u_3 = 6$
Warehouse Demand	200	280	240	360	320	100	1500	
Column Nos. (V_j)	$V_1 = 58$	$V_2 = 48$	$V_3 = 46$	$V_4 = 54$	$V_5 = 54$	$V_6 = 0$		

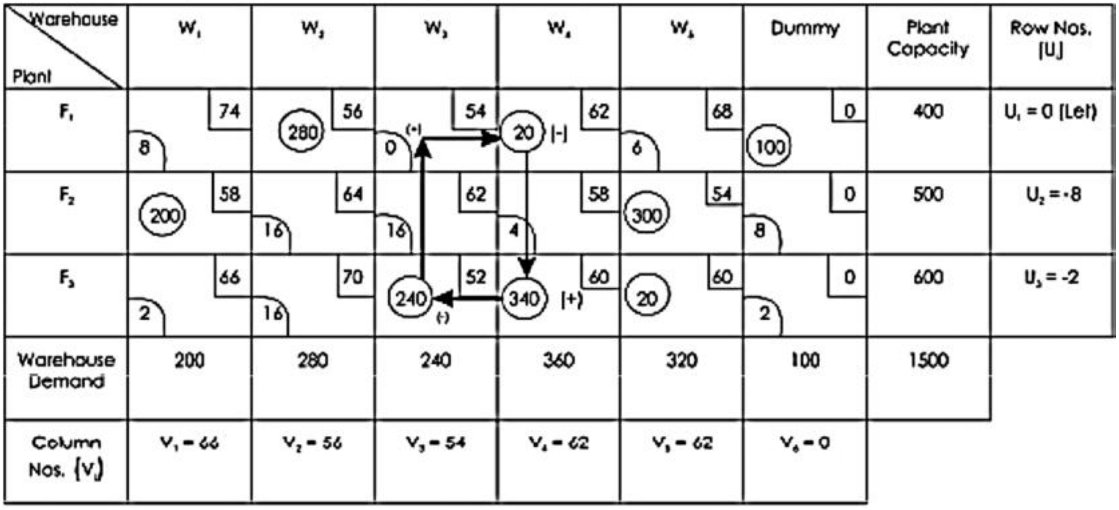
Calculation of Opportunity Costs for Basic Feasible Solution

Unallocated Cell	Opportunity Cost [$\Delta_{ij} = C_{ij} - (u_i + v_j)$]
$F_1 - W_1$	$\Delta_{11} = C_{11} - (u_1 + v_1) = 74 - (8 + 58) = 8$
$F_1 - W_3$	$\Delta_{13} = C_{13} - (u_1 + v_3) = 54 - (8 + 46) = 0$
$F_1 - W_5$	$\Delta_{15} = C_{15} - (u_1 + v_5) = 68 - (8 + 54) = 6$
F_1 - Dummy	$\Delta_{16} = C_{16} - (u_1 + v_6) = 0 - (8 + 0) = - 8$
$F_2 - W_2$	$\Delta_{22} = C_{22} - (u_2 + v_2) = 64 - (0 + 48) = 6$
$F_2 - W_3$	$\Delta_{23} = C_{23} - (u_2 + v_3) = 62 - (0 + 46) = 16$
$F_2 - W_4$	$\Delta_{24} = C_{24} - (u_2 + v_4) = 58 - (0 + 54) = 4$
$F_3 - W_1$	$\Delta_{31} = C_{31} - (u_3 + v_1) = 66 - (6 + 58) = 2$
$F_3 - W_2$	$\Delta_{32} = C_{32} - (u_3 + v_2) = 70 - (6 + 48) = 16$
F_3 - Dummy	$\Delta_{36} = C_{36} - (u_3 + v_6) = 0 - (6 + 0) = - 6$

As all the Opportunity Costs are not nonnegative, the solution is non optimal i.e. further improvement is possible.

For this a loop is formed starting from the cell having highest negative value which is cell (F1 - Dummy) having a highest negative opportunity cost value of -8. The starting cell of the loop is marked with a (+) and thereafter alternately the corner cells of the loop are marked (–) and (+). Next the minimum of the allocated quantities of the cells marked (–) is subtracted from the allocated quantities of all the cells marked (–) and added to all the cells marked (+). This leads to an improved solution as shown below.

Table: 3 Showing Improved Solution (Optimal)



Warehouse / Plant	W1	W2	W3	W4	W5	Dummy	Plant Capacity	Row Nos. [U]
F1	74 8	56 280	54 (+) 0	62 20 (-)	68 6	0 100	400	$U_1 = 0$ [Let)
F2	58 200	64 16	62 16	58 4	54 300	0 8	500	$U_2 = -8$
F3	66 2	70 16	52 240 (-)	60 340 (+)	60 20	0 2	600	$U_3 = -2$
Warehouse Demand	200	280	240	360	320	100	1500	
Column Nos. (V)	$V_1 = 66$	$V_2 = 56$	$V_3 = 54$	$V_4 = 62$	$V_5 = 62$	$V_6 = 0$		

Opportunity Costs (Dij) for the unallocated cells are calculated same as before and shown in left bottom corner of the cells.

(a) As Dij ≥ 0, the solution is optimal.

Table -4: Showing Optimal Distribution Plan

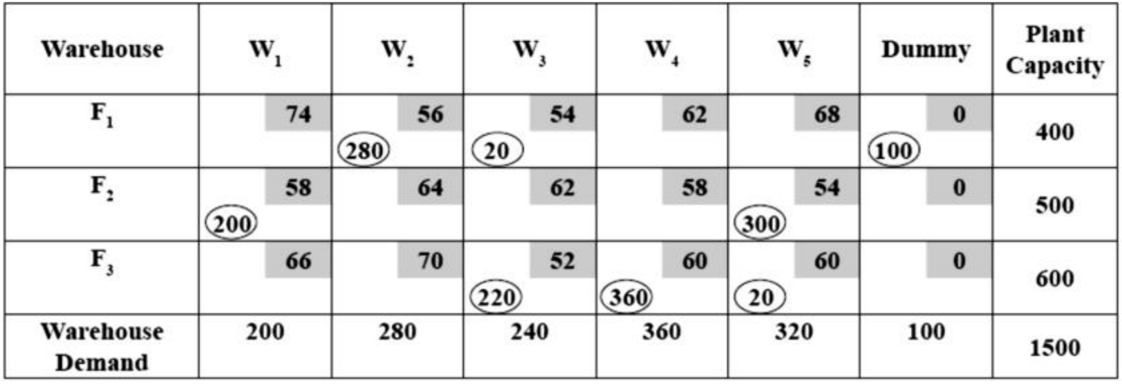
From Plant	To Warehouse	Quantity (Units)	Cost/Unit (Rs.)	Total (Rs.)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
F_1	W_2	280	56	15680
	W_4	20	62	1240
	Dummy	100	0	0
F_2	W_1	200	58	11600
	W_5	300	54	16200
F_3	W_3	240	52	12480
	W_4	340	60	20400
	W_5	20	60	1200
Total		1500	-	Rs. 78800

Minimum Cost of Transportation is Rs. 78800

(b) Plant F1 is having a surplus quantity of 100 units.

(c) Presence of zero opportunity cost (in the cell F1 -W3) indicates that alternative optimum solution is possible for the problem. To get the solution, we form a loop starting from the cell F1 - W3. The new solution is shown below-

Table-5: Showing Alternative Optimum Solution



Warehouse	W_1	W_2	W_3	W_4	W_5	Dummy	Plant Capacity
F_1	74	56 280	54 20	62	68	0 100	400
F_2	58 200	64	62	58	54 300	0	500
F_3	66	70	52 220	60 360	60 20	0	600
Warehouse Demand	200	280	240	360	320	100	1500

Table-6: Showing Alternative Optimum Distribution Plan

From Plant	To Warehouse	Quantity (Units)	Cost/Unit (Rs.)	Total (Rs.)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
F_1	W_2	280	56	15680
	W_3	20	54	1080
	Dummy	100	0	0
F_2	W_1	200	58	11600
	W_5	300	54	16200
F_3	W_3	220	52	11440
	W_4	360	60	21600
	W_5	20	60	1200
Total		**1500**	-	**Rs. 78800**

So the alternative solution is given above.

Illustration 21:

A company has 4 factories F1, F2, F3, & F4 manufacturing the same product. Production & row material cost differ from factory to factory and are given in the following table in the first two rows.

The transportation cost from factories to sales departments S_1, S_2, S_3, are also given. The last two columns in the table give the sales price & the total requirement at each sales department.

The production capacity of each factory is given in the last row.

Factories
Sales Dept.

Factories / Sales Dept.	F_1	F_2	F_3	F_4	Sales price / unit	Requirement
Production cost/unit	15	18	14	13		
Row material cost/unit	10	9	12	9		
Transportation Cost/unit S_1	3	9	5	5	34	80
S_2	1	7	4	5	32	120
S_3	5	8	3	6	31	150
Availability	10	150	50	100		

Determine the most profitable production & the distribution schedule & the corresponding profit. The surplus product should be taken to yield zero profit.

Answer:

Initially in this problem there are four sources (factories) and three destinations (sales Depot)
Total Cost/unit = Production cost/unit + Raw material cost/unit + Transportation Cost/unit.
Profit/unit = Selling Price/unit - (Total Cost/unit)
Total Availability = 310 units & Total requirement = 350 units Since Total Availability not equal to total requirement so it is a unbalanced transportation problem.
Since total availability is less than total requirement we have to introduce a dummy factory with adjustment of 40 units to make balance transportation problem.

Table showing the calculation of per unit Profit matrix

	Sales depot 1	**Sales depot 2**	**Sales depot 3**	**Availability**
Factory 1	34 - (15 + 10 + 3) = 6	32 - (15 + 10 + 1) = 6	1	10
Factory 2	-2	-2	-4	150
Factory 3	3	2	2	50
Factory 4	7	5	3	100
Factory 5 (Dummy)	0	0	0	40
Requirement	80	120	150	350

Table showing the Calculation of per unit cost matrix [Subtracting each element of cost from biggest element here it is '7']

	Sales depot 1	**Sales depot 2**	**Sales depot 3**	**Availability**
Factory 1	1	1	6	10
Factory 2	9	9	11	150
Factory 3	4	5	5	50
Factory 4	0	2	4	100
Factory 5 (Dummy)	7	7	7	40
Requirement	80	120	150	350

Now we can apply VAM to get initial Basic feasible solutions(IBFS)
For optimality solution we will follow two steps (1) Calculation of Row Penalty (ui) and column penalty (vj) by trial and error method on the basis of occupied solution (IBFS) .
Total no of initial basic feasible solutions = m+n-1 but total no of u_i and v_j are m + n so with the help of m+n-1 IBFS we can never solve m+n unknowns so any one of m+n solutions can be solved by trial and error method .
Any one of the u_i or v_j will be zero on the basis of maximum number of occupied cell if no of occupied cells are same for more than one rows or one columns we can consider any of them to maintain the condition of m+n-1.
Let C_{ij} be the cost for occupied cell .Using occupied cell costs and one of the trial solution we can calculate the other row penalties and column penalties.

Where, $C_{ij} = u_i + v_j$

After getting all ui and vj then we calculate the unoccupied cell using the formula given below:

$C_{ij} - (u_i + v_{j)}$

where, C_{ij} is the cost of unoccupied cell.

Illustration 22:

Departmental store wishes to purchase the following quantities of Sprees:

Types of sprees	A	B	C	D	E
Quantity	150	100	75	250	200

Tenders are submitted by 4 different manufacturers who undertake to supply not more than the quantities mentioned below (all types of sprees combined):

Manufacturer	W	X	Y	Z
Total quantity	300	250	150	200

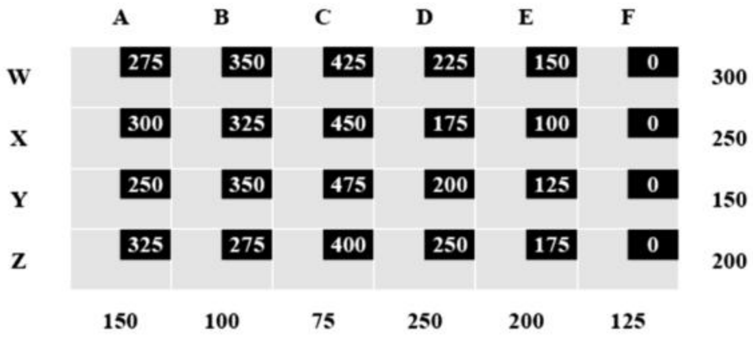
The store estimates that its profit/spree will vary with the manufacturer as shown in the following matrix.

Manufacturers	Sprees				
	A	B	C	D	E
W	275	350	425	225	150
X	300	325	450	175	100
Y	250	350	475	200	125
Z	325	275	400	250	175

How should the orders be placed?

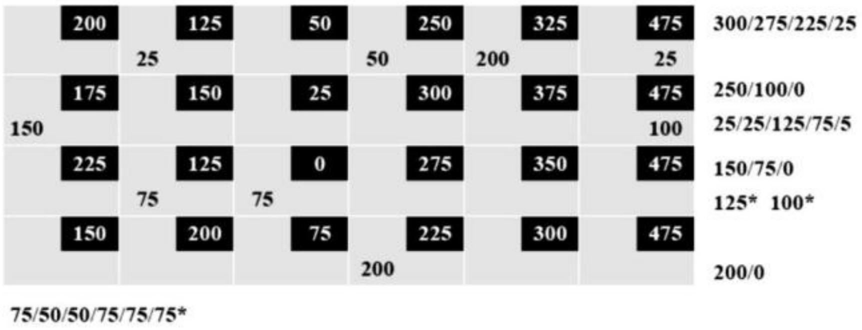
Answer:

Profit matrix:



	A	B	C	D	E	F	
W	275	350	425	225	150	0	300
X	300	325	450	175	100	0	250
Y	250	350	475	200	125	0	150
Z	325	275	400	250	175	0	200
	150	100	75	250	200	125	

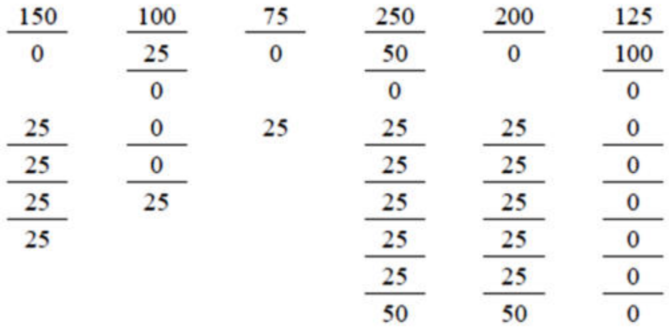
Loss Matrix:



200	125 25	50	250 50	325 200	475 25	300/275/225/25
175 150	150	25	300	375	475 100	250/100/0 25/25/125/75/5
225	125 75	0 75	275	350	475	150/75/0 125* 100*
150	200	75	225 200	300	475	200/0

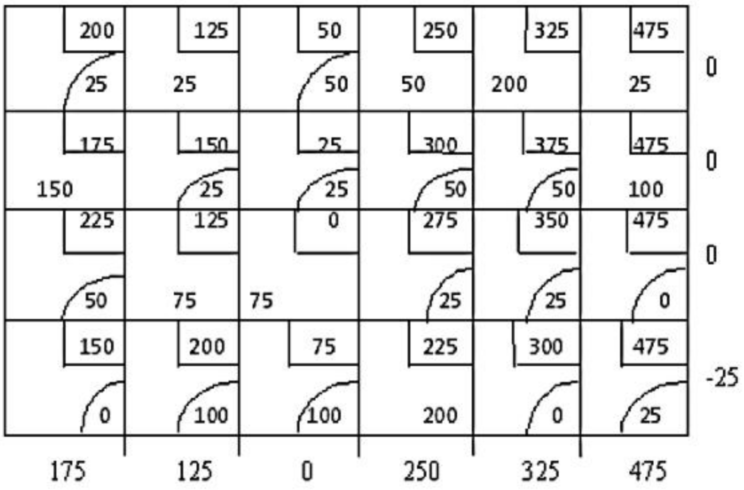
75/50/50/75/75/75*

m + n – 1 allocation s are there, optimality test can be performed.



150	100	75	250	200	125
0	25	0	50	0	100
	0		0		0
25	0	25	25	25	0
25	0		25	25	0
25	25		25	25	0
25			25	25	0
			25	25	0
			50	50	0

M + n – 1 allocation s are there, optimality test can be performed.



200 25	125 25	50 50	250 50	325 200	475 25	0
175 150	150 25	25 25	300 50	375 50	475 100	0
225 50	125 75	0 75	275 25	350 25	475 0	0
150 0	200 100	75 100	225 200	300 0	475 25	-25
175	125	0	250	325	475	

As Δij ≥ 0, maximum profit is as follows.

Qty Maximum Profit

W → B 25 × 350 = 8750

D 50 × 225 = 11250

E 200 × 150 = 30000

F 25 × 0 = 0

X → A 150 × 300 = 45000

F 100 × 0 = 0

Y → B 75 × 350 = 26250

C 75 × 475 = 35625

Z → D 200 × 250 = 50000

Max. Profit. 900 Rs. 2,06,875

Illustration 23:

The Bombay Transport Company has trucks available at four different sites in the following numbers:

Site A	5 Trucks
Site B	10 Trucks
Site C	7 Trucks
Site D	3 Trucks

Customers – W, X and Y require trucks as shown below.

Customer W	5 Trucks
Customer X	8 Trucks
Customer Y	10 Trucks

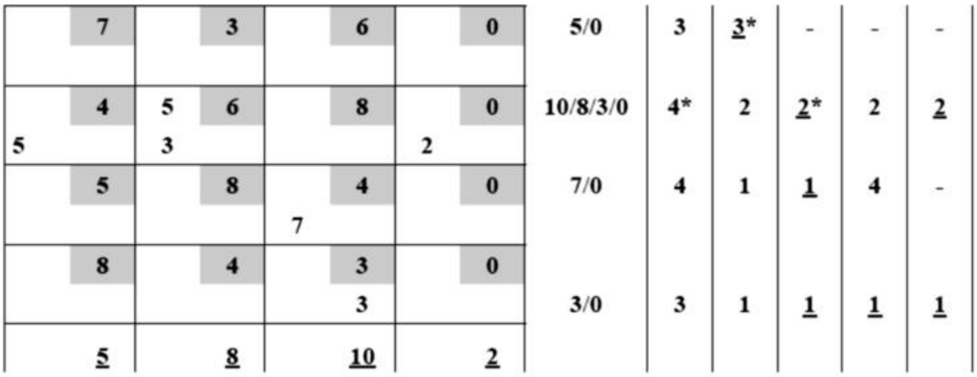
Variable Costs of getting trucks to the Customers are given below:

From A to W	Rs. 7, to X	Rs. 3 to Y	Rs. 6
From B to W	Rs. 4, to X	Rs. 6 to Y	Rs. 8

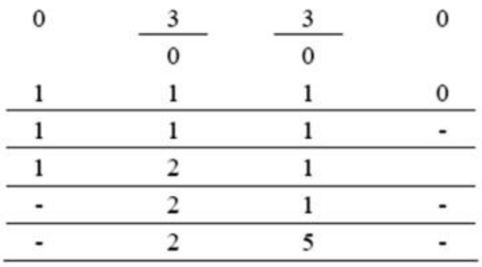
From C to W	Rs. 5, to X	Rs. 8 to Y	Rs. 4
From D to W	Rs. 8 to X	Rs. 4 to Y	Rs. 3

Solve the above transportation problem.

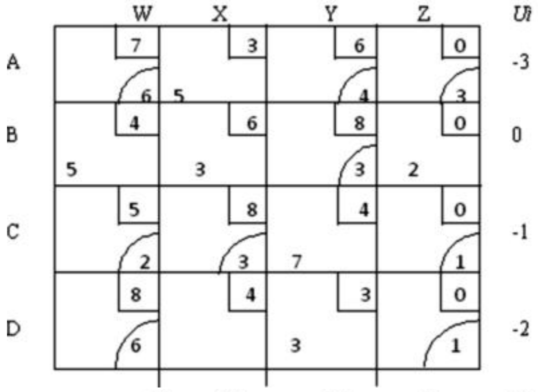
Answer:



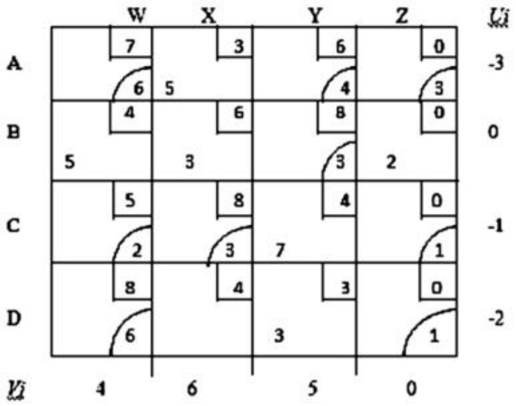
7	3	6	0	5/0	3	3*	-	-	-
4 5	5 6 3	8	0 2	10/8/3/0	4*	2	2*	2	2
5	8	4 7	0	7/0	4	1	1	4	-
8	4	3 3	0	3/0	3	1	1	1	1
5	8	10	2						



0	3	3	0
	0	0	
1	1	1	0
1	1	1	-
1	2	1	
-	2	1	-
-	2	5	-



	W	X	Y	Z	Ui
A	7 6	3 5	6 4	0 3	-3
B	4 5	6 3	8 3	0 2	0
C	5 2	8 3	4 7	0 1	-1
D	8 6	4	3 3	0 1	-2



	W	X	Y	Z	Ui
A	7 6	3 5	6 4	0 3	-3
B	4 5	6 3	8 3	0 2	0
C	5 2	8 3	4 7	0 1	-1
D	8 6	4	3 3	0 1	-2
Vi	4	6	5	0	

As $\Delta_{ij} \geq 0$, the solution is optimum.

Allocation:

Minimum Cost

A →X → 5 × 3 = 15

B →W → 5 × 4 = 20

→ X → 3 × 6 = 18

→ Z → 2 × 0 = 0

C → Y → 7 × 4 = 28

D → Y → 3 × 3 = 9

25 **Rs. 90**

Illustration 24:

A company has 3 plants located at different places but producing an identical product. The cost of production, distribution cost of each plant to the 3 different warehouses, the sale price at each warehouse and the individual capacities for both the plant and warehouse are given below:

Plants	F1	F2	F3		
Raw material	15	18	14		
Other expenses	10	9	12		
Distribution cost to warehouse				Sales Price in (Rs.)	Warehouse Capacity (No)
W1	3	9	5	34	80
W2	1	7	4	32	110
W3	5	8	3	31	150
Capacity of Plant (No.)	150	100	130		

Establish a suitable table giving net profit/loss for a unit produced at different plants and distributed at different locations.

(a) Introduce a suitable dummy warehouse / plant so as to match the capacities of plants and warehouses.

(b) Find distribution pattern so as to maximize profit / minimize loss.

(c) Interpret zero value of square evaluation of an empty cell and find alternative solutions.

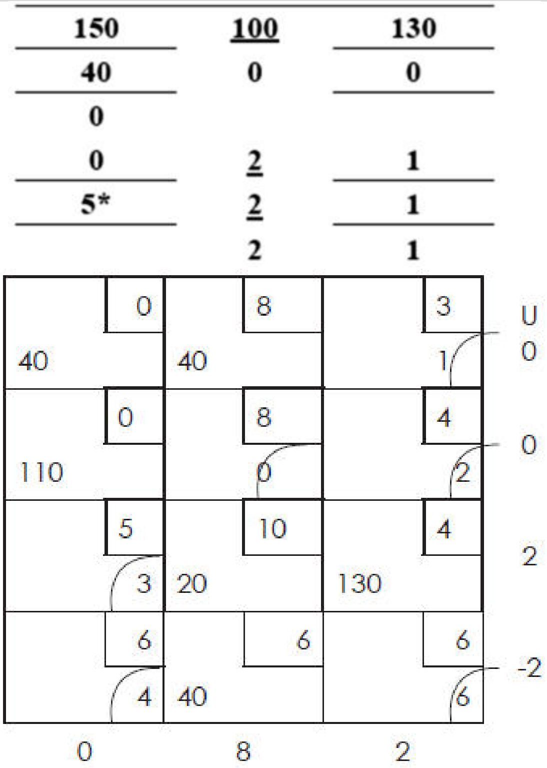
Answer:

Profit matrix

6	-2	3	80
6	-2	2	110
1	-4	2	150
0	0	0	40
150	100	130	380

Loss Matrix:

0 40	8 40	3	80/40/0	3/3/5
0 110	8	4	110/2	4*
5	10 20	4 130	150/20/0	1/1/6*
6	6 40	6	10/0	0/0/0



150	100	130
40	0	0
0		
0	2	1
5*	2	1
	2	1

			U
0 40	8 40	3 1	0
0 110	8 0	4 2	0
5 3	10 20	4 130	2
6 4	6 40	6 6	-2
0	8	2	

As there are m+n-1 allocations, optimality test can be performed since Δij ≥ 0,

		Quantity	**Maximum Profit**
F1	W1	40 × 6	240
	W2	40 × -2	-80
F2	W1	110 × 6	660
F3	W2	20 × -4	-80
	W3	130 × 2	260
F4 Dummy	W2	40 × 0	0
		380	**Rs. 1000**

Profit Rs. 1,000/-

4.6 Job Evaluation, Job Allocation, Assignment

Questions For Classroom Discussion

Assignment Problem

(Hungarian Method)

Different Methods of Solving Assignment Problem

1) Complete Enumeration Method
2) Simplex Method
3) Transportation Problem
4) Hungarian Method

Question 1

Road Construction Project by the Ministry of PWD

The Ministry of Public Works Department (PWD) has undertaken a project to construct four roads. The department has decided to award a maximum of one road to each contractor. The objective is to complete the construction of all roads at the minimum cost within a common timeframe. Four contractors have submitted their tender quotes as detailed below.

	Contractor 1	Contractor 2	Contractor 3	Contractor 4
Road 1	12	30	21	15
Road 2	18	33	9	31
Road 3	44	25	21	21
Road 4	14	30	28	14

As a Cost and Management Accountant student, you can assist the PWD Department in reducing the total cost of constructing all roads by implementing the following strategies:

Question 2

Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows.

	J1	J2	J3	J4	J5
E1	8	4	2	6	1
E2	0	9	5	5	4
E3	3	8	9	2	6
E4	4	3	1	0	3
E5	9	5	8	9	5

Problems of Type 2

Multiple Solutions

Question 3

Given below is the time (days) required when a particular program is assigned to a particular programmer.

	A	B	C	D
P1	12	10	8	9
P2	8	9	11	7
P3	11	14	12	10
P4	9	9	8	9

Question 4

Solve the minimal assignment problem whose effectiveness matrix is

	J1	J2	J3	J4
A	2	3	4	5

B	4	5	6	7
C	7	8	9	8
D	3	5	8	4

1) A > J2, B > J3, C > J4, D > J1
2) A > J3, B > J2, C > J4, D > J1
3) A > J1, B > J2, C > J3, D > J4
4) A > J1, B > J3, C > J2, D > J4
5) A > J2, B > J1, C > J3, D > J4
6) A > J2, B > J3, C > J1, D > J4
7) A > J3, B > J1, C > J2, D > J4
8) A > J3, B > J2, C > J1, D > J4

Drawing Straight Line

Problems of Type 3

Question 5

A project work consists of three major jobs for which three contractors have submitted tenders. The tender amounts quoted in Lakhs of Rupees are given in the matrix below.

	A	B	C
C1	17	25	31
C2	10	25	16
C3	12	14	11

Question 6

Solve the following assignment problem

	J1	J2	J3	J4	J5
A	1	3	2	3	6
B	2	4	3	1	5
C	5	6	3	4	6
D	3	1	4	2	2
E	1	5	6	5	4

Question 7

Solve the following assignment problem for minimizing cost.

	J1	J2	J3	J4
A	32	26	35	38
B	27	24	26	32
C	28	22	25	34
D	10	10	16	16

Maximisation of Assignment Problem

Question 8

Given below is a matrix showing the profit for different jobs done through different machines. Find an assignment program which will maximize the total profit.

	M1	M2	M3	M4
J1	51	53	54	50
J2	47	50	48	50
J3	49	50	60	61

J4	63	64	60	61

Question 9

A college department chairman has the problem of providing instructions for the courses offered by his department at the highest possible level of educational quality. He has arrived at the following relative ratings regarding the ability of each instructor to each of the four courses

	OPM	Costing	Tax	FM
Faculty 1	4	3	2	5
Faculty 2	5	3	2	5
Faculty 3	4	4	3	2
Faculty 4	3	4	5	2

Unbalanced Assignment Problem

Question 10

A company has four machines to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table. What are job assignments, which will minimize the cost?

	J1	J2	J3	J4
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22
	***	***	***	***

Question 11

Solve the maximisation Problem

	A	B	C	D	E
M1	62	78	50	101	82
M2	71	84	61	73	59
M3	87	92	111	71	81
M4	48	64	87	77	80

Prohibited Assignment Problem (Restricted Assignment Problem)

Question 12

In the modification of a plant lay out of a factory four new machine M1, M2,M3, M4 are to be installed in a machine shop. There are five vacant places A,B,C,D, and E available. Because of the limited space machine M2 cannot be placed at C and M3 cannot be placed at A. The cost of locating places to machines is shown below. Find the optimal assignment schedule?

Which space remains vacant after assignment?

	A	B	C	D	E
M1	9	11	15	10	11
M2	12	9	****	10	9
M3	***	11	14	11	7
M4	14	8	12	7	8

Travelling Salesman Problem

Question 13

A salesman has to visit five cities A,B,C,D,E. The distances between cities are given in the table below. If the salesman starts from the City A and has to come back to City A. Which route should he select, so that total distance travelled by him is minimized?

	A	B	C	D	E
A	*	4	7	3	4
B	4	*	6	3	4
C	7	6	*	7	5
D	3	3	7	*	7
E	4	4	5	7	*

Question 14

Solve the following travelling salesman problem.

	A	B	C	D	E
A	***	10	25	25	10
B	1	***	10	15	2
C	8	9	***	20	10
D	14	10	24	***	15
E	10	8	25	27	***

Question 15 **Previous year Question**

A project consists of four major jobs. For which four contractors submitted tenders. The tender amounts, in thousand of Rupees, are given below.

	A	B	C	D
A	110	98	75	95
B	85	95	115	65
C	105	135	125	98
D	95	95	75	95

Question 16 **Previous Year Question – June 2015 – 16 Marks**

A department head has four sub ordinates and four tasks have to be performed. Sub ordinates differ in efficiency and tasks differ in their intrinsic difficulty. Time each man would take to perform each task is given in the effectiveness matrix. How the task would be allotted to each person so as to minimize the total man hours?

	J1	J2	J3	J4
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

The questions from the institute material follow this one. After this question, we have completed the six questions available in the institute material. Please refer to the institute material for further details.

Illustration From Study Material

Illustration 25:

Six men are available for different jobs. From past records the time in hours taken by different persons for different jobs are given below.

	Jobs						
Men	1	2	3	4	5	6	
	1	2	9	2	7	9	1
	2	6	8	7	6	14	1
	3	4	6	5	3	8	1
	4	4	2	7	3	10	1
	5	5	3	9	5	12	1
	6	9	8	12	13	9	1

Find out an allocation of men to different jobs which will lead to minimum operation time.

Answer:

Job / **Man**	**1**	**2**	**3**	**4**	**5**	**6**
1	2	9	2	7	9	1
2	6	8	7	6	14	1
3	4	6	5	3	8	1
4	4	2	7	3	10	1
5	5	3	9	5	12	1
6	9	8	12	13	9	1

Row Operation* (Table - 1)

Job / **Man**	**1**	**2**	**3**	**4**	**5**	**6**
1	1	8	1	6	8	0
2	5	7	6	5	13	0
3	3	5	4	2	7	0
4	3	1	6	2	9	0
5	4	2	8	4	11	0
6	8	7	11	12	8	0

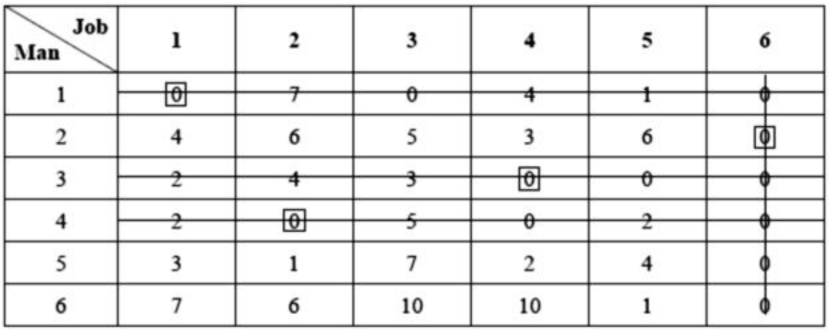
* Matrix is obtained by subtracting min. element of each row of the given Matrix from all the elements of the corresponding row.

Column Operation* (Table - 2)

Job / **Man**	**1**	**2**	**3**	**4**	**5**	**6**
1	0	7	0	4	1	0
2	4	6	5	3	6	0
3	2	4	3	0	0	0
4	2	0	5	0	2	0
5	3	1	7	2	4	0
6	7	6	10	10	1	0

* Matrix is obtained by subtracting min. element of each column of Table - 1 from all the elements of the corresponding column.

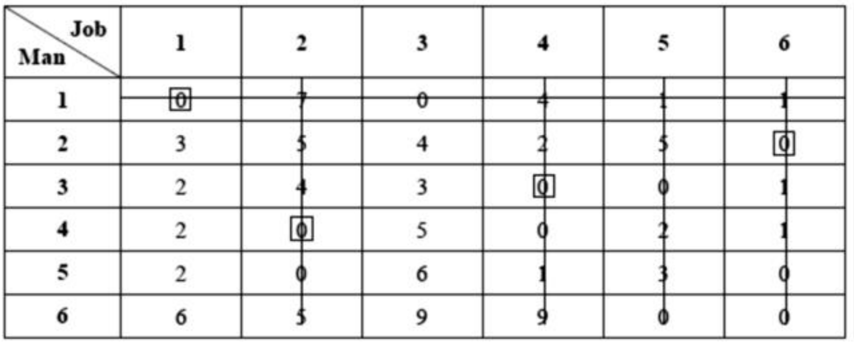
Table - 3



Job Man	1	2	3	4	5	6
1	$\boxed{0}$	7	0	4	1	0
2	4	6	5	3	6	$\boxed{0}$
3	2	4	3	$\boxed{0}$	0	0
4	2	$\boxed{0}$	5	0	2	0
5	3	1	7	2	4	0
6	7	6	10	10	1	0

All the zeros obtained in Table - 2 are covered by minimum no. of horizontal and vertical straight lines and shown above. Here order of the given matrix = 6 and minimum no. of horizontal and vertical lines = 4. As 4 ≠ 6, the solution is non optimal.

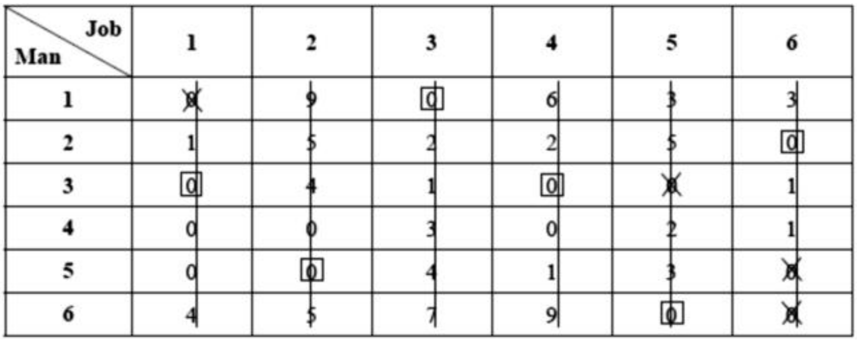
Table - 4



Job Man	1	2	3	4	5	6
1	$\boxed{0}$	7	0	4	1	1
2	3	5	4	2	5	$\boxed{0}$
3	2	4	3	$\boxed{0}$	0	1
4	2	$\boxed{0}$	5	0	2	1
5	2	0	6	1	3	0
6	6	5	9	9	0	0

Above table is obtained by subtracting minimum uncovered element of Table - 3 from all the uncovered elements and by adding the same to all the elements at the junction of the intersecting straight lines. Minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ 6 (order of the matrix). So the solution is non optimal.

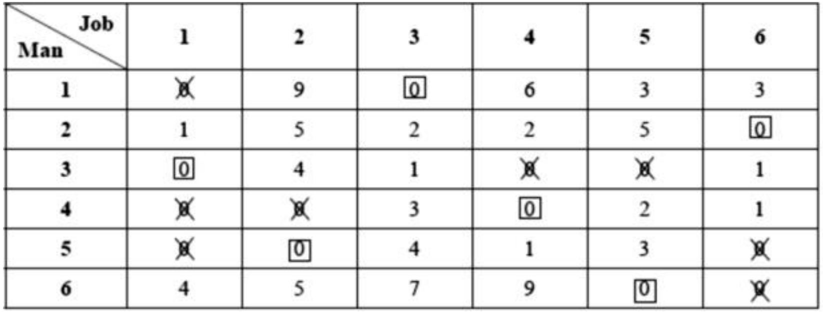
Table - 5



Job Man	1	2	3	4	5	6
1	$\cancel{0}$	9	$\boxed{0}$	6	3	3
2	1	5	2	2	5	$\boxed{0}$
3	$\boxed{0}$	4	1	$\boxed{0}$	$\cancel{0}$	1
4	0	0	3	0	2	1
5	0	$\boxed{0}$	4	1	3	$\cancel{0}$
6	4	5	7	9	$\boxed{0}$	$\cancel{0}$

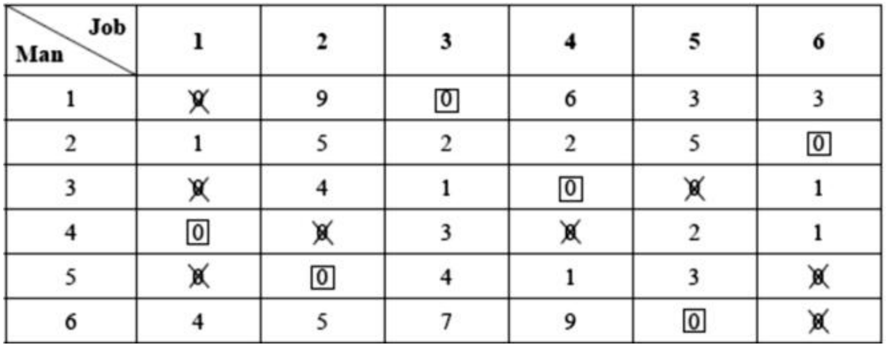
Above table is obtained by subtracting minimum uncovered element (2) of Table - 4 from all the uncovered elements and by adding the same to all the elements at the junction of the intersecting straight lines. Here minimum no. of horizontal or vertical straight lines to cover all the zeros = 6 = Order of the Matrix. So the solution is optimal.

Table - 6 Showing Optimum Solution - 1



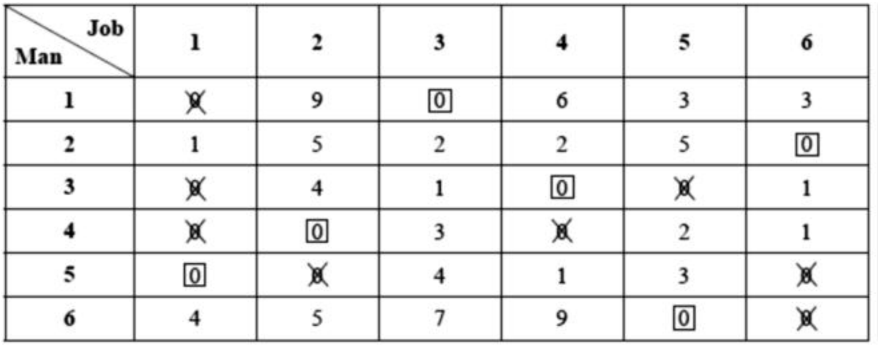
Job Man	1	2	3	4	5	6
1	$\cancel{0}$	9	$\boxed{0}$	6	3	3
2	1	5	2	2	5	$\boxed{0}$
3	$\boxed{0}$	4	1	$\cancel{0}$	$\cancel{0}$	1
4	$\cancel{0}$	$\cancel{0}$	3	$\boxed{0}$	2	1
5	$\cancel{0}$	$\boxed{0}$	4	1	3	$\cancel{0}$
6	4	5	7	9	$\boxed{0}$	$\cancel{0}$

Table - 7 Showing Optimum Solution - 2



Job / Man	1	2	3	4	5	6
1	$\xcancel{0}$	9	$\boxed{0}$	6	3	3
2	1	5	2	2	5	$\boxed{0}$
3	$\xcancel{0}$	4	1	$\boxed{0}$	$\xcancel{0}$	1
4	$\boxed{0}$	$\xcancel{0}$	3	$\xcancel{0}$	2	1
5	$\xcancel{0}$	$\boxed{0}$	4	1	3	$\xcancel{0}$
6	4	5	7	9	$\boxed{0}$	$\xcancel{0}$

Table - 8 Showing Optimum Solution - 3



Job / Man	1	2	3	4	5	6
1	$\xcancel{0}$	9	$\boxed{0}$	6	3	3
2	1	5	2	2	5	$\boxed{0}$
3	$\xcancel{0}$	4	1	$\boxed{0}$	$\xcancel{0}$	1
4	$\xcancel{0}$	$\boxed{0}$	3	$\xcancel{0}$	2	1
5	$\boxed{0}$	$\xcancel{0}$	4	1	3	$\xcancel{0}$
6	4	5	7	9	$\boxed{0}$	$\xcancel{0}$

So the Optimal Assignments are as follows :—

As per Table - 6			As per Table - 7			As per Table - 8		
Man	**Job**	**Time (hrs.)**	**Man**	**Job**	**Time (hrs.)**	**Man**	**Job**	**Time (hrs.)**
1	3	2	1	3	2	1	3	2
2	6	1	2	6	1	2	6	1
3	1	4	3	4	3	3	4	3
4	4	3	4	1	4	4	2	3
5	2	3	5	2	3	5	1	5
6	5	9	6	5	9	6	5	9
Total	–	22	**Total**	–	22	**Total**	–	22

Minimum total operation time = 22 hrs.

Illustration 26:

A captain of a cricket team has to allot five middle batting positions to five batsmen. The average runs scored by each batsman at these positions are as follows:

Batting Position						
Batsmen		III	IV	V	VI	VII
	A	40	40	35	25	50
	B	42	30	16	25	27
	C	50	48	40	60	50
	D	20	19	20	18	25
	E	58	60	59	55	53

Make the assignment so that the expected total average runs scored by these batsmen are maximum.

Answer:

This is a problem of Maximisation. To solve it using Assignment technique it has to be converted to a Minimisation problem by forming a Relative Loss Matrix.

	Batting Position				
Batsman	**III**	**IV**	**V**	**VI**	**VII**
A	40	40	35	25	50
B	42	30	16	25	27
C	50	48	40	60	50
D	20	19	20	18	25
E	58	60	59	55	53

Relative Loss Matrix*

	Batting Position				
Batsman	**III**	**IV**	**V**	**VI**	**VII**
A	20	20	25	35	10
B	18	30	44	35	33
C	10	12	20	0	10
D	40	41	40	42	35
E	2	0	1	5	7

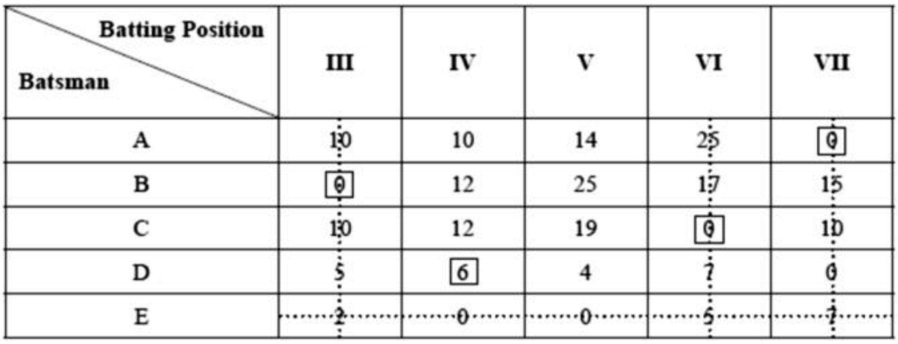
* This matrix is formed by subtracting all the elements of the given matrixfrom the highest element (60) of it.

Row Operation Matrix

	Batting Position				
Batsman	**III**	**IV**	**V**	**VI**	**VII**
A	10	10	15	25	0
B	0	12	26	17	15
C	10	12	20	0	10
D	5	6	5	7	0
E	2	0	1	5	7

Column Operation Matrix

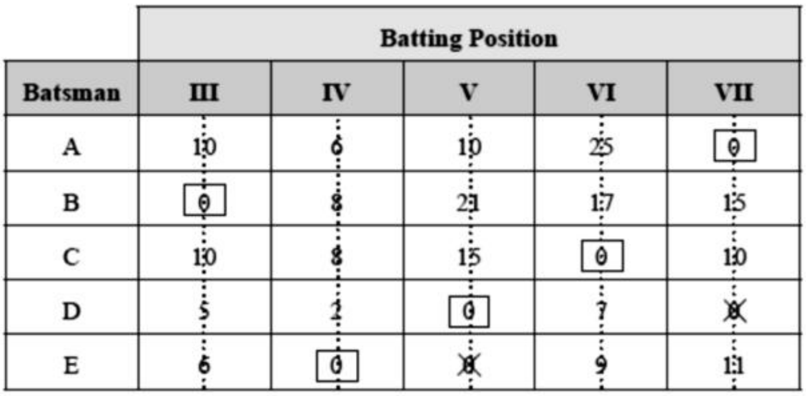
Batting Position



Batting Position / Batsman	III	IV	V	VI	VII
A	10	10	14	25	$\boxed{0}$
B	$\boxed{0}$	12	25	17	15
C	10	12	19	$\boxed{0}$	10
D	5	$\boxed{6}$	4	7	0
E	2	0	0	5	7

Minimum no. of horizontal and vertical straight lines to cover all the zeros = 4 ≠ Order of the matrix(5). So the solution is non optimal.

Improved Matrix



	Batting Position				
Batsman	**III**	**IV**	**V**	**VI**	**VII**
A	10	6	10	25	$\boxed{0}$
B	$\boxed{0}$	8	21	17	15
C	10	8	15	$\boxed{0}$	10
D	5	2	$\boxed{0}$	7	$\cancel{0}$
E	6	$\boxed{0}$	$\cancel{0}$	9	11

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 = Order of the matrix. So the solution is optimal.

Optimal Assignment

Batsman	Batting Position	Average runs scored
A	VII	50
B	III	42
C	VI	60
D	V	20
E	IV	60
	Total =	**232**

Expected maximum total runs = 232

Illustration 27:

Average time taken by an operator on a specific machine is tabulated below. The management is considering replacing one of the old machines by a new one and the estimated time for operation by each operator on the new machine is also indicated.

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6

(a) Find out an allocation of operators to the old machines to achieve a minimum operation time.

(b) Reset the problem with the new machine and find out the allocation of the operators to each machine and comment on whether it is advantageous to replace an old machine to achieve a reduction in operating time only.

(c) How will the operators be reallocated to the machines after replacement?

Answer:

Operator	**Machines**						
	M_1	**M_2**	**M_3**	**M_4**	**M_5**	**M_6**	**New**
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6

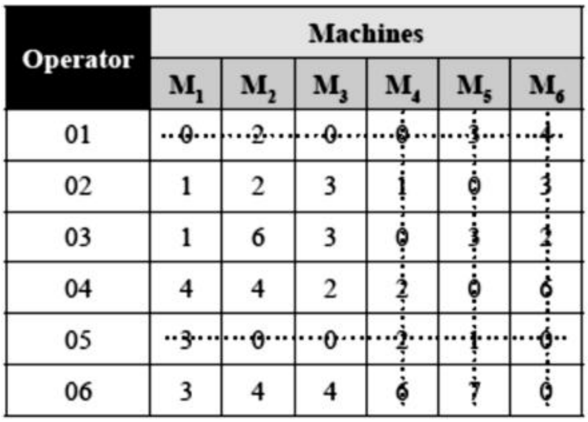
(a) Matrix after Row Operation

Operator	**Machines**					
	M_1	**M_2**	**M_3**	**M_4**	**M_5**	**M_6**
01	2	3	2	1	4	5
02	4	4	6	3	2	5
03	6	10	8	4	7	6
04	8	7	6	5	3	9
05	7	3	4	5	4	3
06	5	5	6	7	8	1

Operator	**Machines**					
	M_1	**M_2**	**M_3**	**M_4**	**M_5**	**M_6**
01	1	2	1	0	3	4
02	2	2	4	1	0	3
03	2	6	4	0	3	2
04	5	4	3	2	0	6
05	4	0	1	2	1	0
06	4	4	5	6	7	0

To find out the allocation of the Old Machines to the operators we consider the given matrix without the new machine.

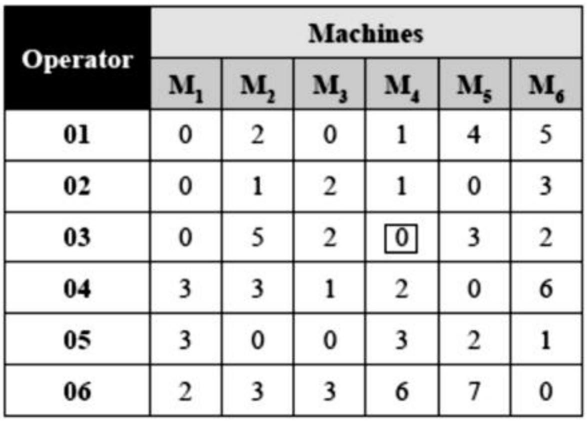
Matrix after Column Operation



Operator	Machines					
	M_1	M_2	M_3	M_4	M_5	M_6
01	0	2	0	0	3	4
02	1	2	3	1	0	3
03	1	6	3	0	3	2
04	4	4	2	2	0	6
05	3	0	0	2	1	0
06	3	4	4	6	7	0

Minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ order of the matrix (6). So the solution is non optimal.

Improved matrix



Operator	Machines					
	M_1	M_2	M_3	M_4	M_5	M_6
01	0	2	0	1	4	5
02	0	1	2	1	0	3
03	0	5	2	$\boxed{0}$	3	2
04	3	3	1	2	0	6
05	3	0	0	3	2	1
06	2	3	3	6	7	0

Minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 = Order of the matrix. So the solution is optimal.

Optimal Assignment

Operators	01	→	M3	-	2
	02	→	M1	-	4
	03	→	M4	-	4
	04	→	M5	-	3
	05	→	M2	-	3
	06	→	M6	-	1

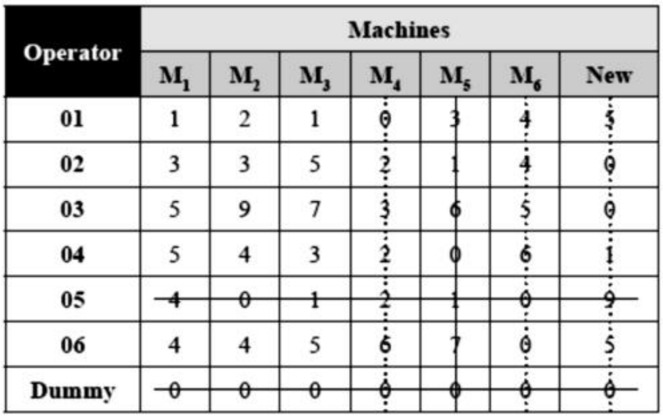
17 Hours Minimum Operation Time

(b) & (c)

Operator	**Machines**						
	M1	**M2**	**M3**	**M4**	**M5**	**M6**	**New**
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6
Dummy	0	0	0	0	0	0	0

With the introduction of a new machine into the system, the problem becomes unbalanced one. To make it balanced, a Dummy operator is introduced and all the elements of the matrix corresponding to it are taken as zero.

(1) Matrix after Row Operation

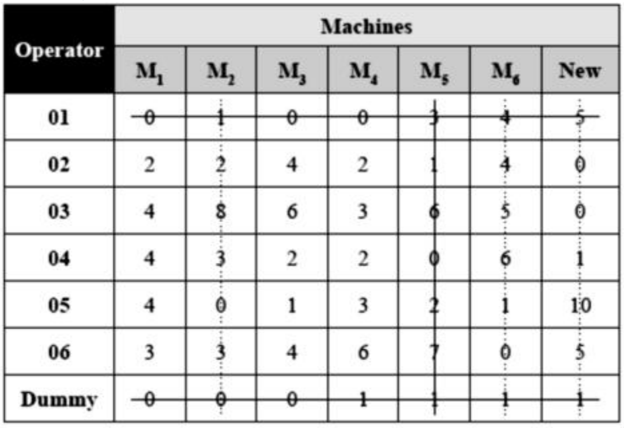


Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	1	2	1	0	3	4	5
02	3	3	5	2	1	4	0
03	5	9	7	3	6	5	0
04	5	4	3	2	0	6	1
05	4	0	1	2	1	0	9
06	4	4	5	6	7	0	5
Dummy	0	0	0	0	0	0	0

As all the columns contain zeros, the matrix after column operation will remain same. So the operation need not be done.

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 ≠ order of the matrix (7). So the solution is non optimal.

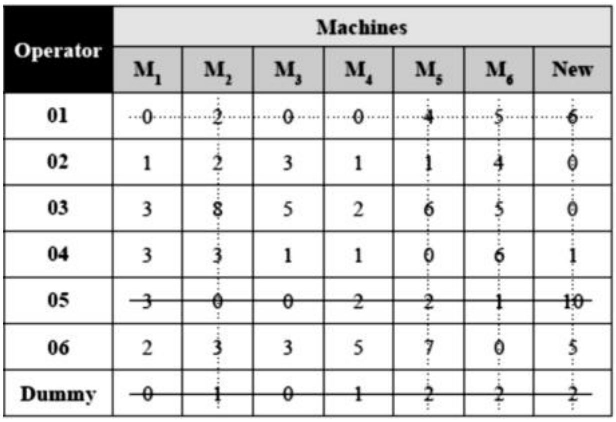
(2) Improved Matrix



Operator	Machines						
	M$_1$	M$_2$	M$_3$	M$_4$	M$_5$	M$_6$	New
01	0	1	0	0	3	4	5
02	2	2	4	2	1	4	0
03	4	8	6	3	6	5	0
04	4	3	2	2	0	6	1
05	4	0	1	3	2	1	10
06	3	3	4	6	7	0	5
Dummy	0	0	0	1	1	1	1

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 ≠ order of the matrix (7). So the solution is non optimal.

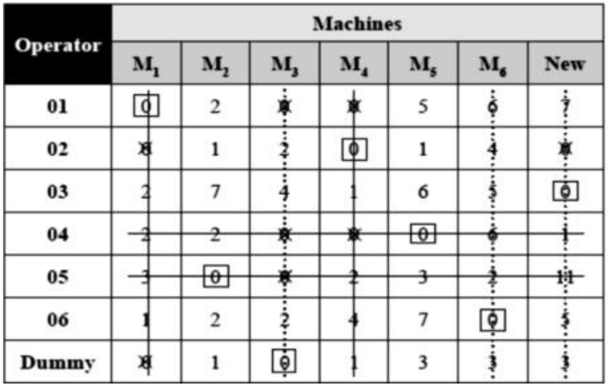
(3) Improved Matrix



Operator	Machines						
	M$_1$	M$_2$	M$_3$	M$_4$	M$_5$	M$_6$	New
01	0	2	0	0	4	5	6
02	1	2	3	1	1	4	0
03	3	8	5	2	6	5	0
04	3	3	1	1	0	6	1
05	3	0	0	2	2	1	10
06	2	3	3	5	7	0	5
Dummy	0	1	0	1	2	2	2

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 ≠ order of the matrix(7). So the solution is non optimal.

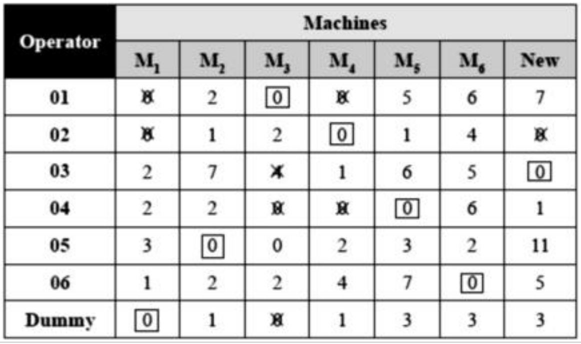
(4) Improved Matrix Showing Optimal Solution (i)



Operator	Machines						
	M$_1$	M$_2$	M$_3$	M$_4$	M$_5$	M$_6$	New
01	$\boxed{0}$	2	0	0	5	6	7
02	0	1	2	$\boxed{0}$	1	4	0
03	2	7	4	1	6	5	$\boxed{0}$
04	2	2	0	0	$\boxed{0}$	6	1
05	3	$\boxed{0}$	0	2	3	2	11
06	1	2	2	4	7	$\boxed{0}$	5
Dummy	0	1	$\boxed{0}$	1	3	3	3

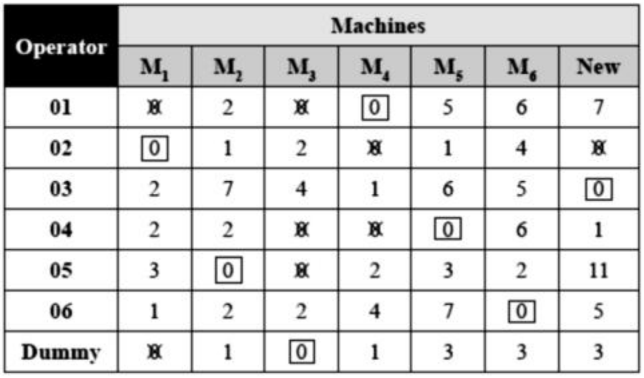
Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 7 = order of the matrix. So the solution optimal.

Improved Matrix Showing Optimal Solution (ii)



Operator	Machines						
	M$_1$	M$_2$	M$_3$	M$_4$	M$_5$	M$_6$	New
01	0	2	$\boxed{0}$	0	5	6	7
02	0	1	2	$\boxed{0}$	1	4	0
03	2	7	0	1	6	5	$\boxed{0}$
04	2	2	0	0	$\boxed{0}$	6	1
05	3	$\boxed{0}$	0	2	3	2	11
06	1	2	2	4	7	$\boxed{0}$	5
Dummy	$\boxed{0}$	1	0	1	3	3	3

Improved Matrix Showing Optimal Solution (iii)



Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	$\cancel{0}$	2	$\cancel{0}$	$\boxed{0}$	5	6	7
02	$\boxed{0}$	1	2	$\cancel{0}$	1	4	$\cancel{0}$
03	2	7	4	1	6	5	$\boxed{0}$
04	2	2	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	6	1
05	3	$\boxed{0}$	$\cancel{0}$	2	3	2	11
06	1	2	2	4	7	$\boxed{0}$	5
Dummy	$\cancel{0}$	1	$\boxed{0}$	1	3	3	3

Table Showing Multiple Optimum Allocations

Solution (i)			Solution (ii)			Solution (iii)		
Operators	M/C	Time (Hrs.)	Operators	M/C	Time (Hrs.)	Operators	M/C	Time (Hrs.)
01	M1	2	01	M3	5	01	M4	1
02	M4	3	02	M4	1	02	M1	4
03	New	1	03	New	6	03	New	1
04	M5	3	04	M5	0	04	M5	3
05	M2	3	05	M2	3	05	M2	3
06	M6	1	06	M6	7	06	M6	1
Dummy	M3	0	Dummy	M1	3	Dummy	M3	0
Total	-	13*	**Total**	-	13*	**Total**	-	13*

* Minimum Operation Time

From above it can be said that replacement of an old machine with the new one will result in a reduction in Total Operating Time by 17-13 = 4 Hours. So replacement decision is advantageous.

As per solutions (i) & (iii) above, Machine M3 should be replaced by a New Machine and as per Solution (iii), M_1 should be replaced by a New one.

Illustration 28:

Six salesmen are to be allocated to six sales regions so that the cost of allocation of the job will be minimum.

Each salesman is capable of doing the job at different cost in each region. The cost matrix is given below:

	Region						
		I	II	III	IV	V	VI
Salesmen	A	15	35	0	25	10	45
	B	40	5	45	20	15	20
	C	25	60	10	65	25	10
	D	25	20	35	10	25	60
	E	30	70	40	5	40	50
	F	10	25	30	40	50	15

(Figures are in Rupees)

(a) Find the allocation to give minimum cost. What is the minimum cost?

(b) Now suppose the above table gives earning of each salesman at each region. How can you find an allocation so that the earning will be maximum? Determine the solution with optimum earning.

(c) There are restrictions for commercial reasons that A cannot be posted to region V and E cannot be posted to region II. Write down the cost matrix suitably after imposing the restrictions.

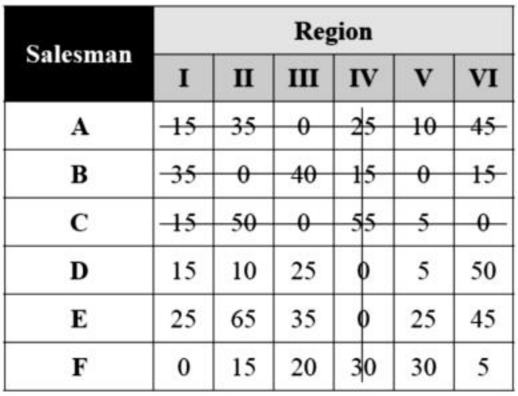
Answer:

Matrix after Row Operation

	Region					
Salesman	**I**	**II**	**III**	**IV**	**V**	**VI**
A	15	35	0	25	10	45
B	40	5	45	20	15	20
C	25	60	10	65	25	10
D	25	20	35	10	25	60
E	30	70	40	5	40	50
F	10	25	30	40	50	15

	Region					
Salesman	**I**	**II**	**III**	**IV**	**V**	**VI**
A	15	35	0	25	10	45
B	35	0	40	15	10	15
C	15	50	0	55	15	0
D	15	10	25	0	15	50
E	25	65	35	0	35	45
F	0	15	20	30	40	5

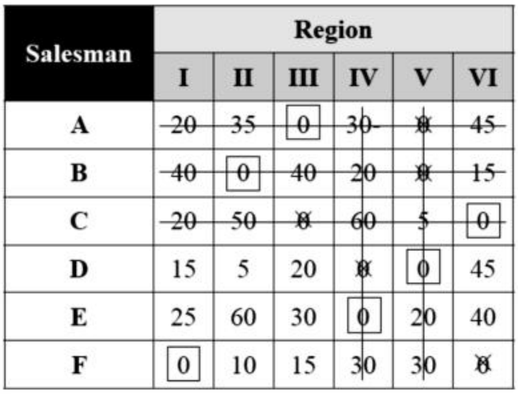
Matrix after Column Operation



Salesman	**Region**					
	I	**II**	**III**	**IV**	**V**	**VI**
A	15	35	0	25	10	45
B	35	0	40	15	0	15
C	15	50	0	55	5	0
D	15	10	25	0	5	50
E	25	65	35	0	25	45
F	0	15	20	30	30	5

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ Order of the matrix (6). So the solution is non optimal.

Improved Matrix (Optimal)



Salesman	**Region**					
	I	**II**	**III**	**IV**	**V**	**VI**
A	20	35	$\boxed{0}$	30	0	45
B	40	$\boxed{0}$	40	20	0	15
C	20	50	0	60	5	$\boxed{0}$
D	15	5	20	0	$\boxed{0}$	45
E	25	60	30	$\boxed{0}$	20	40
F	$\boxed{0}$	10	15	30	30	0

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 = Order of the matrix. So the solution is optimal.

Table showing optimal allocation

Salesman	Region	Cost (Rs.)	
A	III	0	
B	II	5	
C	VI	10	
D	V	25	
E	IV	5	
F	I	10	
Total		**Rs. 55**	Minimum Cost

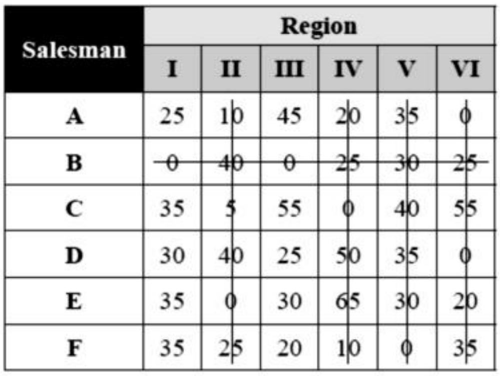
(b) The given problem is a problem of Maximisation. To convert it to a problem of Minimisation, a Relative Loss Matrix is formed by subtracting all the elements of the given matrix from the highest element (70).

Relative Loss Matrix Matrix after Row Operation

	Region					
Salesman	**I**	**II**	**III**	**IV**	**V**	**VI**
A	55	35	70	45	60	25
B	30	65	25	50	55	50
C	45	10	60	5	45	60
D	45	50	35	60	45	10
E	40	0	30	65	30	20
F	60	45	40	30	20	55

	Region					
Salesman	**I**	**II**	**III**	**IV**	**V**	**VI**
A	30	10	45	20	35	0
B	5	40	0	25	30	25
C	40	5	55	0	40	55
D	35	40	25	50	35	0
E	40	0	30	65	30	20
F	40	25	20	10	0	35

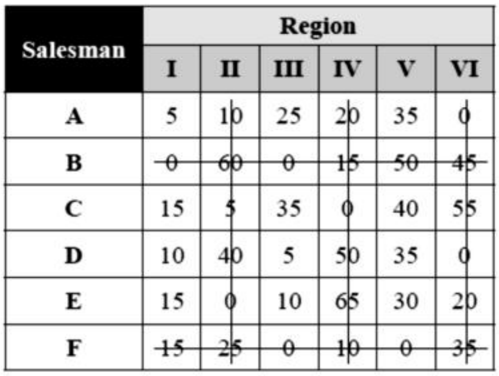
Matrix after Column Operation



Salesman	Region					
	I	II	III	IV	V	VI
A	25	10	45	20	35	0
B	0	40	0	25	30	25
C	35	5	55	0	40	55
D	30	40	25	50	35	0
E	35	0	30	65	30	20
F	35	25	20	10	0	35

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ Order of the matrix (6). So the solution is non optimal.

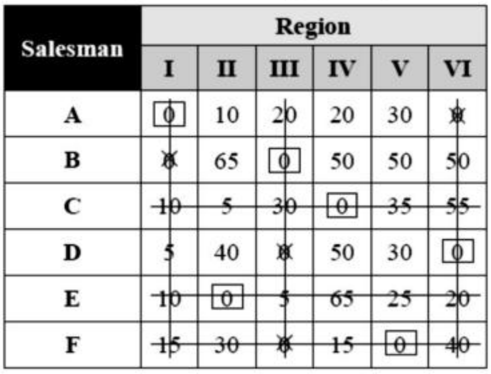
Improved Matrix



Salesman	Region					
	I	II	III	IV	V	VI
A	5	10	25	20	35	0
B	0	60	0	15	50	45
C	15	5	35	0	40	55
D	10	40	5	50	35	0
E	15	0	10	65	30	20
F	15	25	0	10	0	35

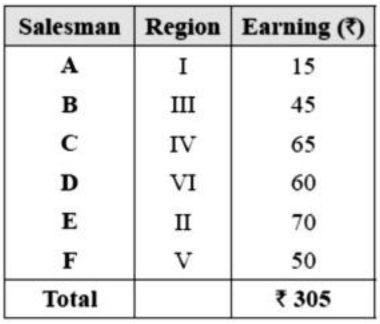
Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ Order of the matrix (6). So the solution is optimal.

Improved Matrix (Optimal)



Salesman	Region					
	I	II	III	IV	V	VI
A	$\boxed{0}$	10	20	20	30	0
B	0	65	$\boxed{0}$	50	50	50
C	10	5	30	$\boxed{0}$	35	55
D	5	40	0	50	30	$\boxed{0}$
E	10	$\boxed{0}$	5	65	25	20
F	15	30	0	15	$\boxed{0}$	40

Table Showing Optimal Allocation



Salesman	Region	Earning (₹)
A	I	15
B	III	45
C	IV	65
D	VI	60
E	II	70
F	V	50
Total		₹ 305

Maximum Earning

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 = Order of the matrix. So the solution is optimal.

(c) The cost matrix after imposing the given restriction is

Region

		I	II	III	IV	V	VI
Sales man	A	15	35	0	25	a	45
	B	40	5	45	20	15	10
	C	25	60	10	65	25	10
	D	25	20	35	10	25	60
	E	30	a	40	5	40	50
	F	10	25	30	40	50	15

Cost (figures are in Rs.)

(Whenever such restrictions are imposed, we have to consider the corresponding element of the given matrix as infinitely large i.e. a)

Illustration 29:

Four jobs can be processed on four different machines, with one job on one machine. Resulting profits vary with assignments. They are given below:

Machines					
		A	B	C	D
Jobs	I	42	35	28	21
	II	30	25	20	15
	III	30	25	20	15
	IV	24	20	16	12

Find the optimum assignment of jobs to machines and the corresponding profit.

Answer:

Relative Loss Matrix

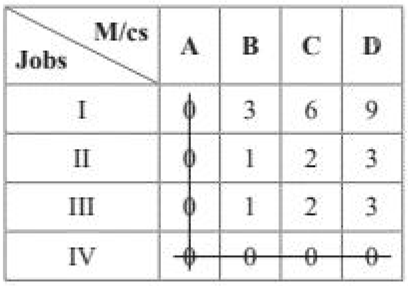
Jobs \ M/cs	A	B	C	D
I	0	7	14	21
II	12	17	22	27
III	12	17	22	27
IV	18	22	26	30

As this is a problem of Maximization, the same is converted to one of Minimization by firming a Relative Loss Matrix where all the elements of the given matrix are subtracted from the highest element of the matrix (which is 42 in this case)

Matrix after Row Operation

Jobs \ M/cs	A	B	C	D
I	0	7	14	21
II	0	5	10	15
III	0	5	10	15
IV	0	4	8	12

Matrix after Column Operation

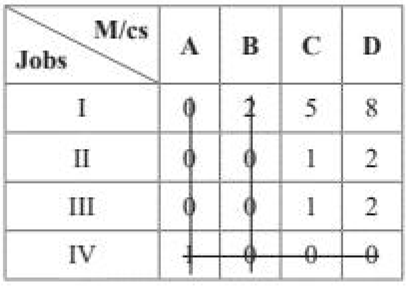


Jobs \ M/cs	A	B	C	D
I	0	3	6	9
II	0	1	2	3
III	0	1	2	3
IV	0	0	0	0

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 2 ≠ Order of the matrix (4)

So the solution is non optimal.

Improved Matrix (Non Optimal)

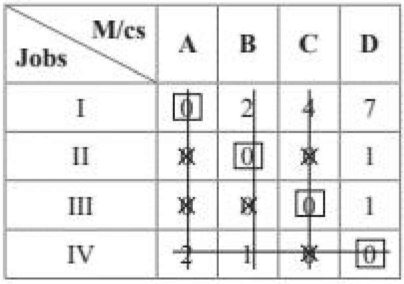


Jobs \ M/cs	A	B	C	D
I	0	2	5	8
II	0	0	1	2
III	0	0	1	2
IV	1	0	0	0

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 3 ≠ Order of the matrix (4)

So the solution is non optimal.

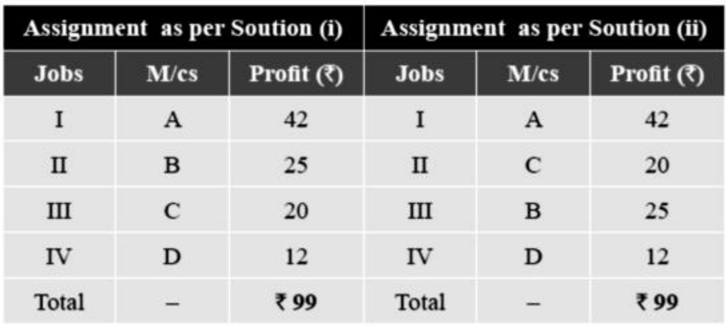
Further Improved Matrix [Optimal Solution (i)]



Jobs \ M/cs	A	B	C	D
I	$\boxed{0}$	2	4	7
II	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	1
III	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	1
IV	2	1	$\cancel{0}$	$\boxed{0}$

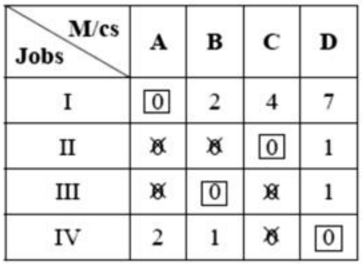
Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 4 = Order of the matrix.

So the solution is optimal.



Assignment as per Soution (i)			Assignment as per Soution (ii)		
Jobs	M/cs	Profit (₹)	Jobs	M/cs	Profit (₹)
I	A	42	I	A	42
II	B	25	II	C	20
III	C	20	III	B	25
IV	D	12	IV	D	12
Total	–	₹ 99	Total	–	₹ 99

Further Improved Matrix
(Optimal Solution-ii)



Jobs \ M/cs	A	B	C	D
I	$\boxed{0}$	2	4	7
II	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	1
III	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	1
IV	2	1	$\cancel{0}$	$\boxed{0}$

Maximum Profit Rs. 99

Illustration 30:

A salesman has to visit five cities A, B, C, D and E. The inter-city distances are tabulated below. Note the distance between two cities need not be same both ways.

From / To	A	B	C	D	E
A	-	12	24	25	15
B	6	--	16	18	7
C	10	11	--	18	12
D	14	17	22	--	16
E	12	13	23	25	--

Note further that the distances are in km.

Required:

If the salesman starts from city A and has to come back to city A, which route would you advise him to take so that total distance traveled by him is minimized?

Answer:

To From	A	B	C	D	E
A	-	12	24	25	15
B	6	-	16	18	7
C	10	11	-	18	12
D	14	17	22	-	16
E	12	13	23	25	-

Row Operation*

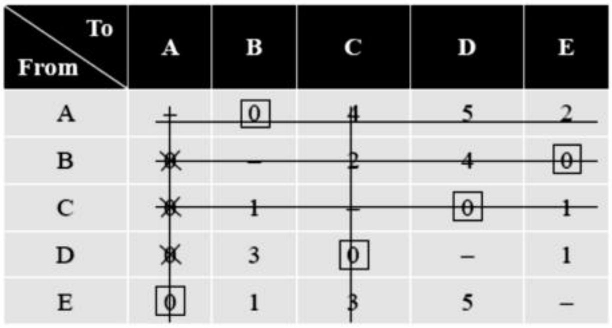
(Table - 1)

To From	A	B	C	D	E
A	-	0	12	13	3
B	0	-	10	12	1
C	0	1	-	8	2
D	0	3	8	-	2
E	0	1	11	13	-

* This matrix is obtained by subtracting minimum element of each row of the given matrix from all the elements of the corresponding row.

Column Operation*

(Table - 2)



To From	A	B	C	D	E
A	–	$\boxed{0}$	4	5	2
B	$\cancel{0}$	–	2	4	$\boxed{0}$
C	$\cancel{0}$	1	–	$\boxed{0}$	1
D	$\cancel{0}$	3	$\boxed{0}$	–	1
E	$\boxed{0}$	1	3	5	–

* This matrix is obtained by subtracting minimum clement of each column of Table-1 from all the elements of the corresponding column.

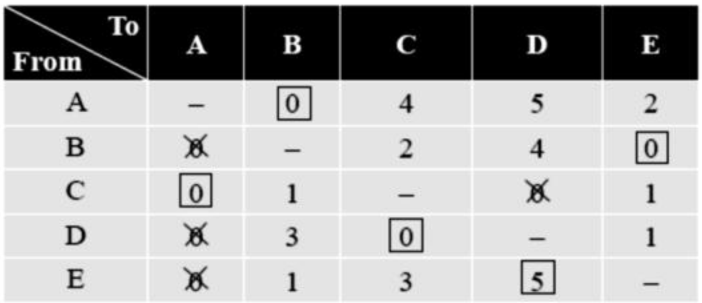
Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 = Order of the matrix.

So the solution is optimal.

Now the solution obtained from the above table shows the travel route of the salesman as A to B, B to E, E to A which means the person is not visiting C and D at all while travelling back to A.

But this is not allowed as per the question.

So the matrix of Table-2 is examined for some of the next best solution which is depicted below.



To From	A	B	C	D	E
A	–	$\boxed{0}$	4	5	2
B	$\cancel{0}$	–	2	4	$\boxed{0}$
C	$\boxed{0}$	1	–	$\cancel{0}$	1
D	$\cancel{0}$	3	$\boxed{0}$	–	1
E	$\cancel{0}$	1	3	$\boxed{5}$	–

Here the assignments have been started by encircling only zero present in the first row which means initial travel route A to B.

Then the only zero present in the last column is encircled which shows subsequent route B to E. Next the only zero of the last row is not encircled because in that case the route would have been E to A which is restricted as per the given condition. So that element of this row is considered which satisfies the restriction. It is 5 indicating the route as E to D. Next the only zero of 3rd column is encircled which means the route as D to C. Finally the only zero row present in the 3rd row is encircled which shows the route as C to A.

Hence the complete route of the Salesman is : A → B → E → D → C → A

Total distance travelled = 12 + 7 + 25 + 22 + 10 = 76 Kms.

This is the optimum distance.

4.7 Scheduling

Questions For Classroom Discussion

Please insert Institute Material Illustration 31 (Scheduling Question, Job, Processing Time, Weight)

Queueing theory

Queuing Theory

Let λ stands for arrival as 'μ' stands for service.

- Probability that service is busy (Not idle) (Percentage of Utilization)

$$=\frac{\lambda}{\mu}$$

- Probability that service is idle (Not Busy) (Percentage of Non Utilization)

$$=1-\left(\frac{\lambda}{\mu}\right)$$

- Probability that 'n' unit in the system

$$\left(\frac{\lambda}{\mu}\right)^{n} 1-\left(\frac{\lambda}{\mu}\right)$$

- Average number of units in the system

$$=\frac{\lambda}{\mu-\lambda}$$

- Average number of units in the queue.

$$=\frac{\lambda^2}{\mu(\mu-\lambda)}$$

- Average time a unit spend in the system.

$$=\frac{1}{\mu-\lambda}$$

- Average time a unit spend in the queue.

$$=\frac{\lambda}{\mu(\mu-\lambda)}$$

- Average length of a non-empty queue.

$$=\frac{\mu}{\mu-\lambda}$$

- Probability of Number of units in the queue is at least 'n'

$$=\left(\frac{\lambda}{\mu}\right)^{n}$$

Question 1

A repair shop attended by a single machine has an average of four customers an hour who bring small appliances. The mechanic inspects them for defects and often can fix them right away or otherwise render a diagnosis. This takes him six minutes, on the average. Arrivals are Poisson and service time has exponential distribution. You are required to

Q.1 Find the probability that shop is empty

Q.2 Find the probability of at least one customer in the shop

Q.3 What is the average number of customers in the system

Q.4 Find the average time spent, including service.

Question 2

In a railway marshalling yard, goods train arrive at a rate of 30 train per day. Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes. Calculate the following

Q.1 Average length of non empty Queue

Q.2 The probability that the Queue size exceeds 10

Question 3

Customers arrive at a booking office window, being manned by a single individual at the rate of 25 per hour. Time required to serve a customer has exponential distribution with a mean of 120 seconds.

Find the average waiting time of customers

Question 4

A TV repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If the repairs set in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day.

Q.1 What is repairman's expected idle time each day ?

Q.2 How many jobs are on an average, ahead of the set just brought in ?

Question 5Previous Year Question- Home Work

The counter of a ration shop experiences the arrival of 25 customers during peak working hours.
Service team will have Poisson distribution. Experiences suggests that means service team should have reached about 2 minutes per customer.

Determine each of the following

1) system utilization
2) Percentage of the time server will be idle
3) Expected number of customers waiting to be served
4) Average time customers will spend in the system

Question 6 Previous Year Question- Home Work

A taxi operator is planning to open a computerised ticket counter in the center of the city, staffed by one ticket agent. It is estimated that requests for ticket and information will average 18 per hour and request will have a Poisson distribution.

Server time is estimated to exponentially distributed. Previous Experience with similar Computerised operations suggests that mean service time should average about 2.5 minutes per request.

Determine each of the following

1) System Utilisation
2) Percentage of time the server (agent) will be idle
3) The expected number of customers waiting to be served
4) The average time customers will spend in the system.

Illustration From Study Material

Illustration 31:

The processing times (tj) in hrs for the five jobs of a single machine scheduling is given. Find the optimal sequence which will minimize the mean flow time and find the mean flow time.

Determine the sequence which will minimize the weighted mean flow time and also find the mean flow time

Job (j)	1	2	3	4	5
Processing time (tj) hrs	30	8	10	28	16
Weight (Wj)	1	2	1	2	3

Answer:

(a) First arrange the jobs as per the shortest processing time (SPT) sequence.

Job (j)	2	3	5	4	1
Processing time (tj) hrs	8	10	16	28	30

Therefore, the job sequence that minimizes the mean flow time is 2-3-5-4-1.

Computation of minimum flow time (F min)

The flow time is the amount of time the job 'j' spends in the system. It is a measure which indicates the waiting of jobs in the system. It is the difference between the completion time (C_j) and ready time (R_j) for job j.

$F_j = C_j - R_j$

Job (j)	2	3	5	4	1
Processing time (t_j) hrs	8	10	16	28	30
Completion time (C_j)	8	18	34	62	92

Since the ready time (Rj) = 0 for all j, the flow time j ü is equal to Cj for all j.

Mean flow time =$(\overline{F})= \frac{1}{n}\sum_{j=1}^{n} F_j = \frac{1}{5}[8+18+34+62+92] = \frac{1}{5}[214] = 42.8$hours

(b) The weights are given as follows:

Job (j)	1	2	3	4	5
Processing time (t_j) hrs	30	8	10	28	16
Weight (W_j)	1	2	1	2	3

The weighted processing time = $\frac{\text{Processing time } (t_j)}{\text{Weight } (W_j)}$

The weighted processing time is represented as

Job (j)	1	2	3	4	5
Processing time (t_j hrs)	30	8	10	28	16
Weight (W_j)	1	2	1	2	3
Weighted Processing time (tj/ W_j)	30	4	10	14	5.31

Thus, arranging the jobs in the increasing order of tj/Wj (weighted shortest processing time WSPT) we have

Job (j)	2	5	3	4	1
Weighted Processing line (t_j/W_j)	4	5.31	10	14	30

optimal sequence that minimizes the weighted mean flow time is 2-5-3-4 -1.

Weighted Mean flow time $(\overline{F}_w) : \overline{F}_w = \frac{\sum_{j=1}^{n} W_j F_j}{\sum_{j=1}^{n} W_j}$

Job (j)	2	5	3	4	1
Processing time (t_j) hrs	8	16	10	28	30
$F_j = (C_j - R_j)$	8	24	34	62	92
W_j	2	3	1	2	1

$F_j \times W_j$	16	72	34	124	92

The weighted mean flow time is computed as follows for optimal sequence.

Weighted mean flow time w ü is computed as

$$\overline{F}_w = \frac{(16 + 72 + 34 + 124 + 92)}{(2 + 3 + 1 + 2 + 1)} = 37.55 \text{ hrs.}$$

Illustration 32:

Customers arrive at a bakery at an average rate of 16 per hour on weekday mornings. The arrival can be described by a Poisson distribution with a mean of 16. Each clerk can serve a customer in an average of three minutes; This time can be described by an exponential distribution with a mean of 3.0 minutes.

a. What are the arrival and service rates?

b. Compute the average number of customers being served at any time.

c. Suppose it has been determined that the average number of customers waiting in line is 3.2. compute the average number of customers in the system (i.e., waiting in line or being served), the average time customers wait in line, and the average time in the system.

d. Determine the system utilization for M = 1, 2, and 3 servers.

Answer:

a. The arrival rate is given in the problem: $\lambda = 16$ customers per hour. Change the service time to a comparable hourly rate by first restating the time in hours and then taking its reciprocal. Thus, (3 minutes per customer)/ (60 minutes per hour) = 1/20 = 1/ μ. Its reciprocal is $\mu = 20$ customers per hour = Service Rate.

b. Average no. of customers being served at any time.

$r = \lambda / \mu = 16 / 20 = 0.80$ customer.

Formulas for basic single-server model

Performance Measure	**Equation**
Average number in line/queue	$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$
Probability of zero units in the system	$P_0 = 1 - \left(\frac{\lambda}{\mu}\right)$
Probability of *n* units in the system	$P_n = P_0 \left(\frac{\lambda}{\mu}\right)^n$
Probability of less than *n* units in the system	$P_{<n} = 1 - \left(\frac{\lambda}{\mu}\right)^n$

c. Given: $L_q = 3.2$ customers

$L_s = L_q + r = 3.2 + 0.80 = 4.0$ customers

Average time customers wait in line

$$= W_q + \frac{L_q}{\lambda} = \frac{3.2}{1} = 0.20 \text{ hour, or } 0.20 \text{ hour} \times 60 \text{ minutes/hour} = 12 \text{ minutes}$$

W_s = Average time customers wait in system = $W_q + \frac{1}{\mu}$

Waiting time in line plus service

$0.20 + \frac{1}{20}$ hour, or 15 minutes

d. System utilization is $\rho = \frac{\lambda}{M \times \mu}$

For M = 1, $\rho = \frac{16}{1(20)} = 0.80$

For M = 2, $\rho = \frac{16}{2(20)} = 0.40$

For M = 3, $\rho = \frac{16}{3(20)} = 0.27$

Note that as the system capacity is measured by Mμ increases, the system utilization for a given arrival rate decreases.

Illustration 33:

An airline is planning to open a satellite ticket desk in a new shopping plaza, staffed by one ticket agent. It is estimated that requests for tickets and information will average 15 per hour, and requests will have a Poisson distribution. Service time is assumed to be exponentially distributed. Previous experience with similar satellite operations suggests that mean service time should average about three minutes per request.

Determine each of the following:

a. System utilization.

b. Percentage of time the server (agent) will be idle.

c. The expected number of customers waiting to be served.

d. The average time customers will spend in the system.

The probability of zero customers in the system and the probability of four customers in the system.

Answer:

Arrival Rate = $\lambda = 15$ customers per hour

Service Rate $= \mu = \frac{1}{\text{Service Time}} = \frac{1 \text{ customer}}{3 \text{ minutes}} \times 60$ minutes per hour = 20 customers per hour

a. System Utilisation $= \rho = \frac{\lambda}{M\mu} = \frac{\lambda}{1(20)} = 0.75$

b. Percentage of time the server will be idle = 1 – r = 1– 0.75 = 0.25, or 25 percent

c. Expected no. of customers waiting to be served $= L_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{225}{20(20-15)} = \frac{225}{(25 \times 5)} =$

$$\frac{225}{100} = 2.25 \text{ customers}$$

d. Average time customers will spend in the system $= W_S = \frac{L_q}{\lambda} + \frac{1}{\mu} = \frac{2.25}{15} + \frac{1}{20} =$

0.20 hours, or 12 minutes

e. Probability of zero customer in the system $= P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{15}{20} = 0.25$ and

Probability of 4 customers in the system $P_4 = P_0 = (\frac{\lambda}{\mu})^4 = 0.25\,(\frac{15}{20})^4 = 0.079$

Illustration 34:

Wanda's Car Wash & dry is an automatic, five-minute operation with a single bay. On a typical Saturday morning, cars arrive at a mean rate of eight per hour, with arrivals tending to follow a Poisson distribution. Find

a. The average number of cars in line.

b. The average time cars spend in line and service.

Answer:

Arrive Rate = $\lambda = 8$ cars per hour

Service Rate = μ = 1 per 5 minutes, or 12 per hour

Av. no. of cars waiting in line $= L_q = \frac{\lambda^2}{2\mu(\mu-\lambda)} = \frac{8^2}{2(12)(12-8)} = 0.667$ car

Av. time cars spend in line and service $= W_S = \frac{L_q}{\lambda} + \frac{1}{\mu} = \frac{0.667}{8} + \frac{1}{12} =$

0.167 hours, or 10 minutes

Illustration 35:

A departmental store has one cashier. During the rush hours, customers arrive at a rate of 20 per hour. The average number of customers that can be handled by the cashier is 24 per hour. Assume the conditions for use of the single – channel queuing model. Find out average time a customer spends in the system.

Answer:

The usual notations are given:

Arrival Rate = λ = 20 customers / hour and service rate = μ = 24 customers / hour

Average no. of customers in the system = $L_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{20}{(24-20)} = \frac{20}{5}$ = 5 customers

Average time a customer spends in the system = $W_S = \frac{L_S}{\lambda} = \frac{5}{20} = \frac{1}{4}$ = 0.25 hour = 15 mins

Illustration 36:

As a tool service centre the arrival rate is two per hour and the service potential is three per hour. Simple queue conditions exist.

The hourly wage paid to the attendant at the service centre is Rs.1.50 per hour and the hourly cost of a machinist away from his work is Rs. 4.

Calculate:

(i) The average number of machinists being served or waiting to be served at any given time.

(ii) The average time a machinist spends waiting for service.

(iii) The total cost of operating the system for an eight – hour day.

(iv) The cost of the system if there were two attendants working together as a team, each paid Rs. 1.50 per hour and each able to service on average 2 per hour.

Answer:

Arrival rate = λ = 2 per hour

Service rate = μ = 3 per hour

(i) Average number of machinists being served or waiting to be served at any given time:

$$L_S = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{2}{(3-2)} = 2$$

(ii) Average Time a machinist spends waiting for the services:

$$W_q = \frac{\lambda}{\mu} \times \frac{1}{(\mu-\lambda)} = \frac{2}{3} \times \frac{1}{(3-2)} = 0.667 \text{ hours}$$

It means a machinist spends 40 minutes (ie., 60 × 0.667) in the queue.

(iii) Average time in the system

$$W_S = \frac{1}{(\mu-\lambda)} = \frac{1}{(3-2)} = 1 \text{ hour}$$

Average number of machinists in the system = 2 [As per (i) above]

Cost of two machinists being away from work = Rs. 4 × 2 = Rs.8.00 per hour

Attendant cost = $\underline{1.50 \text{ per hour}}$

9.50 per hour

Cost of 8- hour day = 8 hrs × Rs.9.50 = Rs.76.00

(iv) It is assumed that there is still a single service point, but the average service rate with 2 attendants now is 4 per hour

∴ Now λ = 2 per hour

m = 4 per hour

∴ Average number of machinists in the system = $L_S = \frac{\lambda}{\mu-\lambda} = \frac{2}{(4-2)} = 1$

Average time spent by a machinist in the system = $W_S = \frac{1}{(\mu-\lambda)} = \frac{1}{(4-2)}$ = 12 hour

Machinists cost = 1/2 hr × Rs.4 =	Rs. 2.00

Attendant cost (@ 1.50 per attendant × 2 attendants	Rs. 3.00
Total Cost	Rs. 5.00

Cost per 8 – hour day = Rs.5 × 8 hrs. = Rs.40.00

Illustration 37:

Workers come to tool store room to enquire about special tools (required by them) for accomplishing a particular project assigned to them. The average time between two arrivals is 60 seconds and the arrivals are assumed to be in Poisson distribution. The average service time (of the tool room attendant) is 40 seconds.

Determine:

(i) average queue length,

(ii) average length of non-empty queues,

(iii) average number of workers in system including the worker being attended

(iv) mean waiting time of an arrival,

(v) average waiting time of an arrival who waits.

Answer:

Here, Arrival Rate = $\lambda = \frac{60}{60}$ per second = 1 per minute

Service Rate = $\mu = \frac{60}{60}$ per second = 1.5 per minute

(i) Average queue length:

$$L_q = \frac{\lambda}{\mu} \times \frac{\lambda}{(\mu - \lambda)} = \frac{1}{1.5} \times \frac{1}{(1.5 - 1)} = \frac{1}{0.75} = \frac{4}{3} \text{ workers}$$

(ii) Average length of non-empty queues:

$$L_n = \frac{\mu}{(\mu - \lambda)} = \frac{1}{(1.5 - 1)} = 3 \text{ workers}$$

(iii) Average number of workers in the system:

$$L_S = \frac{\lambda}{\mu - \lambda} = \frac{1}{(1.5 - 1)} = 2 \text{ workers}$$

(iv) Mean waiting time of an arrival

$$W_q = \frac{\lambda}{\mu} \times \frac{\lambda}{(\mu - \lambda)} = \frac{1}{1.5} \times \frac{1}{(1.5 - 1)} = \frac{3}{4} \text{minutes}$$

(v) Average waiting time of an arrival who waits

$$W_n = \frac{1}{\mu - \lambda} = \frac{1}{(1.5 - 1)} = 2 \text{ minutes}$$

4.8 Simulation and Line Balancing

Questions For Classroom Discussion

Student Hour Required -6

Monte Carlo Simulation

Question 1

A tourist car operator finds that during the past 100 days the demand for the car had been varied as shown below.

Trips Per Day	**0**	**1**	**2**	**3**	**4**	**5**
Number of Days	8	12	15	30	20	15

Using Random numbers, Simulate the demand for a 10 day period
Use Random Numbers 09, 54, 42, 01, 80, 06, 26, 57, 79, 52

Question 2

A confectioner sells confectionery items. Past data of demand per week in hundred kilograms with frequency is given below:

Demand/ Week	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Using the following sequence of random numbers, generate the demand for the next 10 weeks.
Also find out the average demand per week
The random Numbers are 35, 52, 13, 90, 23, 73, 34, 57, 35 83, 94, 56, 67, 66, 60.

Question 3

A trader buys a commodity at Rs. 40 per kg and sells at Rs. 60 per Kg. If any of the commodities remain at the end of the day, it has no saleable value. The loss through unsatisfied demand is Rs. 16 per Kg. Records of the past 250 trading days shows the following.

Supply		Demand	
Quantity Supplied	No of Days of Supply	Quantity Demanded	No of Days of Supply
10	20	10	25
20	25	20	55
30	95	30	100
40	75	40	50
50	35	50	20

- Find net profit in 6 days
- Given the following random numbers 31, 18, 63, 84, 15, 79, 07, 32, 43, 75, 81, 27. Use random numbers alternatively. I.e. First pair 31 to simulate supply, second pair 18 to simulate demand and so on.

Question 4 (Home Work)

A retailer deals in a perishable commodity. The daily demand and supply are variables. The data for the past 500 days show the following demand and supply:

Availability (Supply)		**Demand**	
Availability (kg)	Supply (No of Days)	Demand (Kg)	Demand (No of Days)
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

- The retailer buys the commodity at Rs. 20 per kg. and sells at Rs. 30 per kg. Any commodity remains at the end of the day, has no sales value. Moreover, the loss on unsatisfied demand is Rs. 8 per Kg.
- Given the following pair of random numbers, simulate 6 days sales, demand and profit: (31, 18) (63, 84) (15, 79) (07, 32) (43, 75) (81, 27). The first random number in the pair is that of supply and the second random number is for demand.

After this question, students can attempt Illustrations 40 and 44 as a homework assignment. Hints for solving these illustrations are provided in the upcoming videos.

Question 5

At a service station a study was made over a period of 25 days to determine both the number of automobiles being brought in for service and the number of automobiles serviced. The results are given below.

Number of Auto Mobiles arriving and Servicing	Frequency of Arrival	Frequency of Daily Serviced
0	2	3
1	4	2
2	10	12
3	5	3
4	3	4
5	1	1

- Simulate the arrival and service pattern for a ten-day period and estimate the mean number of automobiles that remain in service for more than a day.
- Use random numbers 09, 54, 42, 01, 80, 06, 06, 26, 67, 79, 49, 16, 36, 76, 68, 91, 97, 85, 56, 84. Use first 10 days for arrival and next 10 days for serviced.

After this question, students can attempt Illustrations 41 as a homework assignment. Hints for solving this illustration are provided in the upcoming videos.

Question 6

An international tourist company deals with numerous personal calls each day and prides itself on its level of service.

The time to deal with each caller depends on the client’s requirements which ranges from, say, a request for a brochure to booking around the world cruise.

If a client has to wait for more than 10 minutes for attention. It is company’s policy for the manager to see him personally and give him a holiday voucher worth Rs. 15.

The company’s observations have shown that the time taken to deal with the clients and arrival pattern of their calls follow the following distribution pattern.

Time to Deal With Clients	Minutes	2	4	6	10	14	20	30
	Probability	0.05	0.10	0.15	0.30	0.25	0.10	0.05
Time Between Arrival	Minutes	1	8	15	25			
	Probability	0.20	0.40	0.30	0.10			

Required:

Q.1 Describe, how would you simulate the operation of the travel agency based on the use of random number tables.

Q.2 Simulate the arrival and serving of 12 clients and show the number of clients who receive a voucher (Use random number below to derive the arrival pattern and line 2 for serving time)

Random numbers

Line 1	03	47	43	73	86	36	96	47	36	61	46	98
Line 2	63	71	62	33	26	16	80	45	60	11	14	10

After this, please proceed with Illustration 43. The logic of the question remains the same; however, in this case, the starting time is taken as 8:00 AM instead of 0. Similar questions involving the arrival and servicing of 12 clients have been tested approximately four times in the CMA Final Strategic Cost Management exam, with marks ranging from 10 to 16. Simplified versions of these questions, involving the arrival and servicing of 5 clients, have been tested at the intermediate level for seven marks.

All the remaining illustrations are discussed in the subsequent videos. There is a 95% chance that questions for the exam will be tested from the first 6 questions we discussed.

4.8 Line Balancing

Questions For Classroom Discussion

Question 1

Table shows the time remaining (number of days until due date) and the work remaining (number of day's still required to finish the work) for 5 jobs which were assigned the letters A to E as they arrived to the shop. Sequence these jobs by priority rules viz.,

a. FCFS – First Come First Served
b. LCFS – Last Come First Served
c. EDD – Early Due Date
d. LS – Least Slack
e. CR- Critical Ratio
f. SPT – Shortest Processing Time
g. LPT - Longest Processing Time

Job	Number of Days Until Due Date	Number of Days of Work Remaining
A	8	7
B	3	4
C	7	5
D	9	2
E	6	6

The following jobs have to be shipped a week from now. The week is having 5 Working Days

Job	A	B	C	D	E	F
Number of days of Work Remaining	2	4	7	6	5	3

Sequence the jobs according to priority rule

A) Least Slack (LS)
B) Critical Ratio (CR)

Question 2

In a factory, there are six jobs to perform, each of which should go through two machines A and B, in the order AB. The processing timings in hours for the jobs are given here. You are required to determine the sequence for performing the jobs that would minimize the total elapsed time, T.

What is the value of T?

Job	Machine A	Machine B
1	7	3
2	4	8
3	2	6
4	5	6
5	9	4
6	8	1

Question 3 **Home Work- Previous Year Question 5 Marks**

A company plans to fill four positions and it decides to conduct attitude test and interviews for the same. While the aptitude tests are conducted by the people from the clerical posts, the job interviews are held by the personal from the management cadre. The job interviews immediately follow the aptitude test. The time required in minutes by each positions are given here

Position	P1	P2	P3	P4
Aptitude Test	100	110	140	120
Job Interview	70	90	80	110

If it is desired to minimise the waiting time of the management personnel, in what order the position filling be handled?

Illustration From Study Material

Illustration 38:

State the major two reasons for using simulation to solve a problem

A confectioner sells confectionery items. Past data of demand per week in hundred kilograms with frequency is given below:

Demand/Week	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Using the following sequence of random numbers, generate the demand for the next 10 weeks. Also find out the average demand per week.

Random numbers	35	52	13	90	23	73	34	57
	35	83	94	56	67	66	60	

Answer:

Table - I

Random No. Range Table for demand

Demand per week	**Frequency (f)**	**Probability (p = f ÷ Σf)**	**Cumulative Probability**	**Range† of Random Nos.**
0	2	.04	.04	00-03
5	11	.22	.26	04-25
10	8	.16	.42	26-41
15	21	.42	.84	42-83
20	5	.10	.94	84-93
25	3	.06	1.00	94-99
	Σf = 50	**1.00**		

†As the given Random Nos. are of 2 digits, the ranges of Random Nos. has also been considered to have 2 digits only. Also the range of Random Nos. corresponds to cumulative probability values which lies between 0 & 1 and can be correlated as nos. between 00 and 99.

Table - II

Simulated Values for next 10 weeks

Weeks	**Random Nos.**	**Demand**
1	35*	10*
2	52	15
3	13	5
4	90	20
5	23	5
6	73	15
7	34	10
8	57	15
9	35	10
10	83	15
Total	-	**120**

*From Table (I), Random No. 35 appears in the range of 26-41. Also the demand for this range is 10.

Average weekly demand = 120 / 10 = 12

Illustration 39:

The manager of a book store has to decide the number of copies of a particular tax law book to order. A book costs Rs. 60 and is sold for Rs. 80. Since some of the tax laws change year after year, any copies unsold while the edition is not current must be sold for Rs. 30. From past records, the distribution of demand for this book has been obtained as follows:

Demand (No of copies)	15	16	17	18	19	20	21	22
Proportion	0.05	0.08	0.20	0.45	0.10	0.07	0.03	0.02

Using the following sequence of random numbers, generate the demand for 20 time periods(years). Calculate the average profit obtainable under each of the courses of action open to the manager. What is the optimal policy?

14	02	93	99	18	71	37	30	12	10
88	13	00	57	69	32	18	08	92	73

Answer:

Random No. Range Table			
Demand	**Probability**	**Cumulative Probability**	**Random No. Range**
15	.05	.05	00-04
16	.08	.13	5-12
17	.20	.33	13-32
18	.45	.78	33-77
19	.10	.88	78-87
20	.07	.95	88-94
21	.03	.98	95-97
22	.02	1.00	98-99
Total	**1.00**	-	-

Calculation of demand and profit for next 20 years					
Year	**Random Numbers**	**Expected demand**	**No. of books unsold if stock is**		
			16*	**17***	**18***
1	14	17	-	-	1
2	02	15	1	2	3
3	93	20	-	-	-
4	99	22	-	-	-
5	18	17	-	-	1
6	71	18	-	-	-
7	37	18	-	-	-
8	30	17	-	-	1
9	12	16	-	1	2
10	10	16	-	1	2
11	88	20	-	-	-
12	13	17	-	-	1
13	00	15	1	2	3
14	57	18	-	-	-
15	69	18	-	-	-
16	32	17	-	-	1
17	18	17	-	-	1
18	08	16	-	1	2
19	92	20	-	-	-
20	73	18	-	-	-

Total	**2**	**7**	**18**

*Looking at the simulated demand pattern, these stock figures have been chosen to find out optimal course of action. Stock figures of 20 or more have not been considered because it is quite obvious that such figures will not give optimal course of action due to high losses for the unsold books.

Statement Showing Computation of Profit			
No. of Books order (n)	**No. of Books sold in 20 years (n × 20 – Books unsold)**	***Net Profit (Rs.)**	**Average Profit/Year (Profit ÷ 20)**
15	15 × 20 = 300	Rs. 6000	Rs. 300
16	16 × 20 – 2 = 318	Rs. 6300 (318 × 20) – 2 × 30	Rs. 315
17	(17 × 20) – 7 = 333	Rs. 6450 (333 × 20) -7 × 30	Rs. 322.5
18	(18 × 20) – 18	Rs. 6300 (342 × 20) – 18 × 30	Rs. 315

* Net Profit = No. of books sold × Rs. 20# – No. of books unsold × Rs. 30**

Selling price/book = Rs. 80, Cost/book = Rs. 60

Profit /book = 80 – 60 = Rs. 20

Selling price of any unsold book = Rs. 30

**Loss incurred/unsold book = Rs. 60 – Rs. 30 = Rs. 30

Since profit is maximum for 17 books order, the optimal policy is to order 17 books per year.

Illustration 40:

A Small retailer has studied the weekly receipts and payments over the past 200 weeks and has developed the following set of information:

Weekly Receipts (Rs.)	Probability	Weekly Payments (Rs.)	Probability
3000	0.20	4000	0.30
5000	0.30	6000	0.40
7000	0.40	8000	0.20
12000	0.10	10000	0.10

Using the following set of random numbers, simulate the weekly pattern of receipts and payments for the 12 weeks of the next quarter, assuming further that the beginning bank balance is Rs. 8000. What is the estimated balance at the end of the 12-weekly period? What is the highest weekly balance during the quarter? What is the average weekly balance for the quarter?

Random Numbers

For Receipts	03	91	38	55	17	46	32	43	69	72	24	22
For payments	61	96	30	32	03	88	48	28	88	18	71	99

According to the given information, the random number interval is assigned to both the receipts and the payments.

Answer:

Range of random numbers							
Receipt (Rs.)	**Probability**	**Cumulative probability**	**Range**	**Payments (Rs.)**	**Probability**	**Cumulative probability**	**Range**
3000	0.20	0.20	00-19	4000	0.30	0.30	00-29
5000	0.30	0.50	20-49	6000	0.40	0.70	30-69
7000	0.40	0.90	50-89	8000	0.20	0.90	70-89
12000	0.10	1.00	90-99	10000	0.10	1.00	90-99

Simulation of Data for a period of 12 weeks					
Week	**Random No. for receipt**	**Expected Receipt (Rs.)**	**Random No. for payment**	**Expected Payment (Rs.)**	**Week end Balance (Rs.)**
Opening Balance					**8000**
1	03	3000	61	6000	5000 (8000 + 3000 – 6000)
2	91	12000	96	10000	7000
3	38	5000	30	6000	6000
4	55	7000	32	6000	7000
5	17	3000	03	4000	6000
6	46	5000	88	8000	3000
7	32	5000	48	6000	2000
8	43	5000	28	4000	3000
9	69	7000	88	8000	2000
10	72	7000	18	4000	5000
11	24	5000	71	8000	2000
12	22	5000	99	10000	(3000)

Estimated balance at the end of 12th week = Rs. (3,000)

Highest balance = Rs. 7,000

Average balance during the quarter = 45,000/12 = Rs. 3,750

Illustration 41:

An automobile production line turns out about 100 cars a day, but deviations occur owing to many causes. The production is more accurately described by the probability distribution given below:

Production/Day	Prob.	Production/Day	Prob.
95	0.03	101	0.15
96	0.05	102	0.10
97	0.07	103	0.07
98	0.10	104	0.05
99	0.15	105	0.03
100	0.20		
		Total	1.00

Finished cars are transported across the bay, at the end of each day, by ferry. If the ferry has space for only 101 cars, what will be the average number of cars waiting to be shipped, and what will be the average number of empty space on the boat? Use following Random Numbers to simulate the data provided above - 20, 63, 46, 16, 45, 41, 44, 66, 87, 26, 78, 40, 29, 92, 21.

Answer:

Simulation of data of an Automobile Production line			
Production/day	**Probability**	**Cumulative Probability**	**Random No. Range**
95	0.03	0.03	00-02
96	0.05	0.08	03-07
97	0.07	0.15	08-14
98	0.10	0.25	15-24
99	0.15	0.40	25-39
100	0.20	0.60	40-59
101	0.15	0.75	60-74
102	0.10	0.85	75-84
103	0.07	0.92	85-91

104	0.05	0.97	92-96
105	0.03	1.00	97-99
	1.00		

Simulated data				
Day	**Random No.**	**Production**	**No.of cars waiting to be shipped**	**No. of empty space on the boat**
1	20	98	-	3
2	63	101	-	-
3	46	100	-	1
4	16	98	-	3
5	45	100	-	1
6	41	100	-	1
7	44	100	-	1
8	66	101	-	-
9	87	103	2	-
10	26	99	-	2
11	78	102	1	-
12	40	100	-	1
13	29	99	-	2
14	92	104	3	-
15	21	98	-	3
Total			6	18

Average no. of cars waiting to be shipped = 6/15 = 0.40 per day

Average no. of empty space on the boat = 18/15 = 1.2 per day

Illustration 42:

A book store wishes to carry 'Ramayana' in stock. Demand is probabilistic and replenishment of stock takes 2 days (i.e. if an order is placed on March 1, it will be delivered at the end of the day on March 3). The probabilities of demand are given below:

Demand (daily)	0	1	2	3	4
Probability	0.05	0.10	0.30	0.45	0.10

Each time an order is placed, the store incurs an ordering cost of Rs. 10 per order. The store also incurs a carrying cost of Rs. 0.50 per book per day. The inventory carrying cost in calculated on the basis of stock at the end of each day.

The manager of the bookstore wishes to compare two options for his inventory decision.

A. Order 5 books when the inventory at the beginning of the day plus order outstanding is less than 8 books.

B. Order 8 books when the inventory at the beginning of the day plus order outstanding is less than 8.

Currently (beginning 1st day) the store has a stock of 8 books plus 6 books ordered two days ago and expected to arrive next day.

Using Monte-Carlo Simulation for 10 cycles, recommend, which option the manager, should choose.

The two-digit random numbers are given below:

89	34	70	63	61	81	39	16	13	73

Answer:

Demand	**Probability**	**Cumulative Probability**	**Random No. Range**
0	0.05	0.05	00-04
1	0.10	0.15	05-14

2	0.30	0.45	15-44
3	0.45	0.90	45-89
4	0.10	1.00	90-99

Option - A

Day	Random No.	Demand	Opening Stock	Ordered Quantity receipt	Closing Stock	Quantity for which Order Placed
1	89	3	8	-	5	-
2	34	2	5	6	9	-
3	70	3	9	-	6	0
4	63	3	6	-	3	5
5	61	3	3	0	0	-
6	81	3	0	5	2	5
7	39	2	2	-	0	5
8	16	2	0	5	3	-
9	13	1	3	5	7	-
10	73	3	7	-	4	5
					39	

Ordering cost 4 × 10	Rs. 40
Carrying cost 0.5 × 39	Rs. 19.50
Total Cost	Rs. 59.50

Option B

Day	R No.	Demand	Opening Stock	Ordered Quantity receipt	Closing Stock	Quantity for which Order placed
1	89	3	8	-	5	-
2	34	2	5	6	9	-
3	70	3	9	-	6	-
4	63	3	6	-	3	8
5	61	3	3	-	0	-
6	81	3	0	8	5	-
7	39	2	5	-	3	8
8	16	2	3	-	1	-
9	13	1	1	8	8	-
10	73	3	8	-	5	-
					45	

Ordering cost 2 × 10	Rs. 20.0
Carrying cost 0.5 × 45	Rs. 22.50
Total Cost	Rs. 42.50

Option 'B' is better because it has low Inventory cost.

Illustration 43:

After observing heavy congestion of customers over a period of time in a petrol station, Mr. Petro has decided to set up a petrol pump facility on his own in a nearby site. He has compiled statistics relating to the potential customer arrival pattern and service pattern as given below. He has also decided to evaluate the operations by using the simulation technique.

Arrivals		Services	
Inter-arrival time (minutes)	Probability	Service time (minutes)	Probability
2	0.22	4	0.28
4	0.30	6	0.40
6	0.24	8	0.22
8	0.14	10	0.10
10	0.10		

Assume:

(i) The clock starts at 8:00 hours

(ii) Only one pump is set up.

(iii) The following12 Random Numbers are to be used to depict the customer arrival pattern:
78, 26, 94, 08, 46, 63, 18, 35, 59, 12, 97 and 82.

(iv) The following 12 Random Numbers are to be used to depict the service pattern:
44, 21, 73, 96, 63, 35, 57, 31, 84, 24, 05, 37

You are required to find out the

(i) probability of the pump being idle, and

(ii) Average time spent by a customer waiting in queue.

Answer:

Inter-arrival time				**Service time**			
Minutes	**Probability**	**Cumulative probability**	**Range of Random No.**	**Minutes**	**Probability**	**Cumulative probability**	**Range**
2	0.22	0.22	00-21	4	0.28	0.28	00-27
4	0.30	0.52	22-51	6	0.40	0.68	28-67
6	0.24	0.76	52-75	8	0.22	0.90	68-89
8	0.14	0.90	76-89	10	0.10	1.00	90-99
10	0.10	1.00	90 - 99	-	-	-	-

Sl.No.	**Random No. for inter arrival time**	**Inter arrival time (Mins.)**	**Entry time in queue as per clock**	**Service start time as per clock**	**Random no for service time**	**Service time (Mins.)**	**Service end time as per clock**	**Waiting time of customer (Mins.)**	**Idle time (Mins.)**
1	78	8	8.08	8.08	44	6	8.14	-	8
2	26	4	8.12	8.14	21	4	8.18	2	-
3	94	10	8.22	8.22	73	8	8.30	-	4
4	08	2	8.24	8.30	96	10	8.40	6	-
5	46	4	8.28	8.40	63	6	8.46	12	-
6	63	6	8.34	8.46	35	6	8.52	12	-
7	18	2	8.36	8.52	57	6	8.58	16	-
8	35	4	8.40	8.58	31	6	9.04	18	-
9	59	6	8.46	9.04	84	8	9.12	18	-
10	12	2	8.48	9.12	24	4	9.16	34	-
11	97	10	8.58	9.16	05	4	9.20	18	-
12	82	8	9.06	9.20	37	6	9.26	14	-
Total Time								**150**	**12**

Average time spent by the customer waiting in the queue = 150/12 = 12.50 minutes

Probability of idle time of petrol station = Total idle time / Total Operating = 12/86 = 0.1395 time of the Service Channel*

*Service End Time – 9.26 Hrs. Service Channel opened at 8.00 hrs. i.e. Total Time of the Service Channel = 1 hr. 26 Mins = 86 Mins.

Illustration 44:

A retailer deals in a perishable commodity. The daily demand and supply are variables. The data for the past 500 days show the following demand and supply:

Availability (Kg.)	Supply (No. of days)	Demand (Kg.)	Demand (No. of days)
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The retailer buys the commodity at Rs. 20 per kg. and sells at Rs. 30 per kg. Any commodity remains at the end of the day, has no sales value. Moreover, the loss on unsatisfied demand is Rs. 8 per Kg. Given the following pair of random numbers, simulate 6 days sales, demand and profit: (31, 18) (63, 84) (15, 79) (07, 32) (43, 75) (81, 27). The first random number in the pair is that of supply and the second random number is for demand.

Answer:

Table-1: Probability Distribution (Supply)

Supply	**Probability**	**Cum. Prob.**	**Range**	**Range of Random Nos. for simulation**
10	40/500 = 0.08	0.08	0 - 0.08	00 - 07
20	50/500 = 0.10	0.18	0.08 - 0.18	08 - 17
30	190/500 = 0.38	0.56	0.18 - 0.56	18 - 55
40	150/500 = 0.30	0.86	0.56 - 0.86	56 - 85
50	70/500 = 0.14	1.00	0.86 - 1.00	86 - 99

Table-2: Probability distribution (Demand)

Demand	**Probability**	**Cum. Prob.**	**Range**	**Range of Random Nos. for simulation**
10	50/500 = 0.10	0.10	0 - 0.10	00 - 09
20	110/500 = 0.22	0.32	0.10 – 0.32	10 – 31
30	200/500 = 0.40	0.72	0.32 - 0.72	32 – 71
40	100/500 = 0.20	0.92	0.72 – 0.92	72 – 91
50	40/500 = 0.08	1.00	0.92 – 1.00	92 – 99

Table-3: Showing simulated data

Simulated data for supply			**Simulated data for demand**		
Day	**Random No.**	**Supply (Kg.)**	**Day**	**Random No.**	**Demand (Kg.)**
1	31	30	1	18	20
2	63	40	2	84	40
3	15	20	3	79	40
4	07	10	4	32	30
5	43	30	5	75	40
6	81	40	6	27	20

Table-4: Statement Showing Supply, Demand and Profit

Day	Supply	Demand	*Sales Revenue	Cost (II)	Loss due to unsatisfied demand (III)	Profit (Rs.)
(a)	(b)	(c)	(d)	(e) = (b) × Rs.20/kg	(f) = [(c)-(b)]× Rs.8/kg	(g) = (d)-(c)-(f)
1	30	20	600	600	-	Nil
2	40	40	1,200	800	-	400
3	20	40	600	400	160	40
4	10	30	300	200	160	-60**
5	30	40	900	600	80	220
6	40	20	600	800	-	-200**

* (1) Sales revenue = Demand × Selling price, when Demand < Supply

(2) Sales revenue = Supply × Selling price, when Demand > Supply

** Negative figures indicate loss

Illustration 45:

Using empirical data A process planner is working on plans for producing a new detergent. She wishes to simulate a raw material demand in order to plan for adequate materials – handing and storage facilities. On the basis of usage for a similar product introduced previously, she has developed a frequency distribution of demand in tons per day for a 2-month period. Use this data (shown below) to simulate the raw material usage requirements for 7 periods (days).

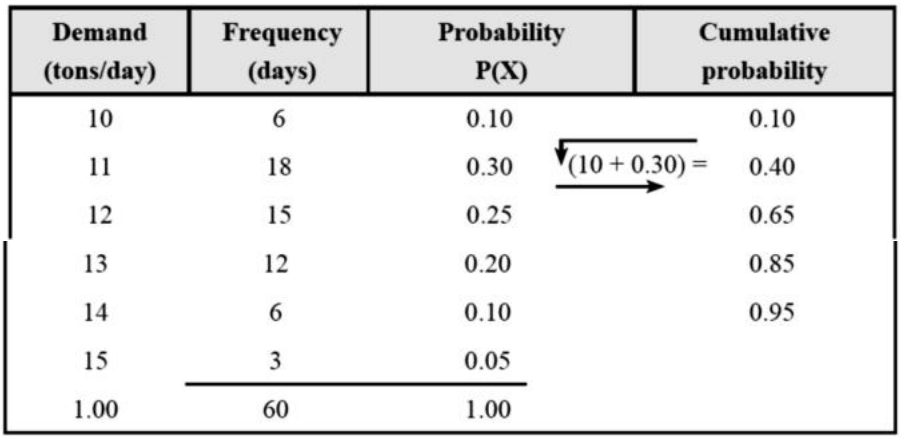
Demands, X (tons/ day)	10	11	12	13	14	15	Total = 60
Frequency (days)	6	18	15	12	6	3	

Answer:

The steps below correspond to those in Fig. 5-17.

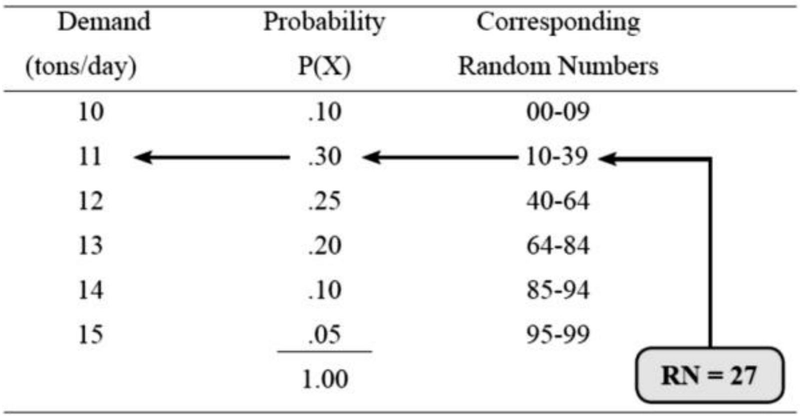
(1) Data are given in frequencies.

(2) To formulate a probability distribution, divide each frequency by the total (60), for example, 6 60 = .10 and 18 60 = .30. Then formulate a cumulative probability distribution by successively summing the probability values.



Demand (tons/day)	Frequency (days)	Probability P(X)	Cumulative probability
10	6	0.10	0.10
11	18	0.30	(10 + 0.30) = 0.40
12	15	0.25	0.65
13	12	0.20	0.85
14	6	0.10	0.95
15	3	0.05	
1.00	60	1.00	

(3) Next, assign random – number intervals so that the number of values available to each class corresponds with the probability. Using 100 two – digit numbers (00-99), we assign 10 percent (00-09) to the first class, 30 percent (10-39) to the second class, and so on.



Demand (tons/day)	Probability P(X)	Corresponding Random Numbers
10	.10	00-09
11 ←	.30 ←	10-39 ←
12	.25	40-64
13	.20	64-84
14	.10	85-94
15	.05	95-99
	1.00	

(4) We obtained random numbers (RN) from column 1 of Appendix I (for convenience), so the first seven numbers are:

27 13 80 10 54 60 49

The first RN, 27, falls into the second class of the distribution and corresponds to a demand of 11 tons per day.

Random Number	27	13	80	10	54	60	49
Simulated Demand	11	11	13	11	12	12	12

(5) This extremely small simulation yields a mean of X = 11.7 tons and a standard deviation of s = .76 tons. The expected value from the empirical probability distribution is E(X) = [XP(X)] = 12.05 tons, suggestion that the small sample size of only 7 periods has resulted in some error. A much larger sample should be simulated before the simulation results are used for making decisions.

Note that the width of the random number "target" in each class corresponds exactly to the relative frequency of the class. This helps to ensure that the simulated results have the same type of distribution as the original data. This is more apparent in the graphic method where the vertical distances on the graph correspond to the relative frequencies of the respective classes.

Illustration 46:

Empirical data collected on the time required to weld a transformer bracket were recorded to the nearest $\frac{1}{4}$ minute, as shown in the accompanying table.

Weld Time (min)	Numbers of Observation
< .25	0
.25 < .75	24
.75 < 1.25	42
1.25 < 1.75	72
1.75 < 2.25	38
2.25 < 2.75	14
2.75 < 3.25	10

(a) Formulate a cumulative distribution in percentage terms.

(b) Graphs the frequency and cumulative distributions.

(c) A simulation is to be conducted using random numbers. What simulated weld times (to the nearest .25 minute) would result from the random numbers 25, 90, and 59?

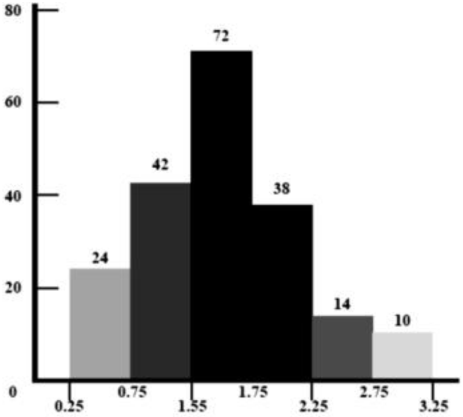
(d) What proportion of the times exceed 2.0 minutes?

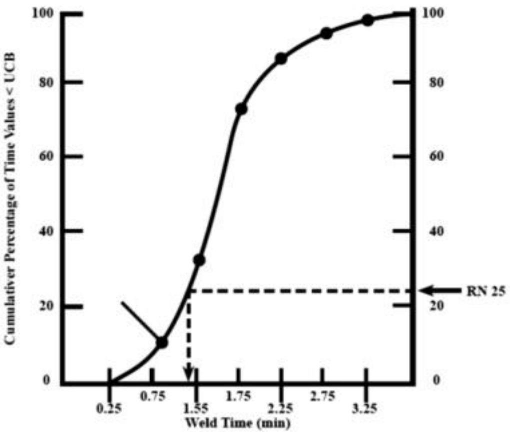
Answer:

(a) Cumulative distributions are usually formulated on a scale where the cumulative percentage is "more than" or "less than" a corresponding X axis amount. We shall use a "less than" percentage and so will need to identify the upper- class boundaries (UCB) as the Y coordinates for the cumulative distribution.

Weld Time (Min)	Frequency In Numbers	Upper – Class Boundary (UCB)	Cumulative Number Of Times < UCB	Cumulative Percentage Of Time < UCB
< .25	0	.25	0	0
.25 < .75	24	.75	24	12
.75 < 1.25	42	1.25	66	33
1.25 < 1.75	72	1.75	138	69
1.75 < 2.25	38	2.25	176	88
2.25 < 2.75	14	2.75	190	95
2.75 < 3.25	10	3.25	200	100

(b) The frequency distribution is constructed by extending vertical lines from the class boundaries to the appropriate frequency level for the class. For the cumulative distribution, values of the cumulative percentage of time < UCB are plotted at weld times corresponding to the UCB. For example, the frequency (12 percent) is plotted at UCB = .75 (as illustrated below).





(c) The simulated time for random number (RN) 25 is determined by entering the cumulative graph at 25 (as shown by the arrow) and proceeding horizontally to the curve and then down to the weld time. The resultant is a reading of 1.0 minute (rounded to the nearest. 25 minutes). Times for random number 90 and 59 are 2.5 and 1.5 minutes, respectively. (A larger graph would lend more accuracy.)

(d) From the cumulative distribution, about 12 percent of the times exceed 2.0 minutes

Illustration 47:

How simulated times can be used to gain a knowledge of the interface of two assembly activitiesIn an aircraft assembly operation, activities A precedes activity B, and inventory may accumulate between the two activities.

With the use of random numbers, a simulated sample of performance times yielded the values shown (minutes) in the accompanying table.

Activity A		Activity B	
Random Number	Time (min)	Random Number	Time
07	.3	63	.5
90	.8	44	.4
02	.2	30	.4
50	.5	98	.9
76	.6	30	.4
47	.5	72	.6
13	.3	58	.5
06	.3	96	.9
79	.7	37	.4

(a) Simulated the assembly of six parts, showing idle time in activity B, waiting time of each part, and number of parts waiting. Note: omit the first random number of A so that activity B begins at time zero.

(b) What was the average length of the waiting line ahead of B (in number of units)?

(c) What was the average output per hour of the assembly line?

Answer:

(a) Our interest lies in activity b, so we can set up a table (below) to show when parts arrive at B, how long it takes B, how long it takes B to work on them, and the resultant idle and waiting times:

Part Number	**Part Available for Activity B at Time**	**Activity B Beginning Time**	**Activity B Ending Time**	**Activity B Idle Time**	**Waiting Time of Part**	**Number parts Waiting at B End time**
1	-	0	.5	0	0	0
2	.8	.8	1.2		.3	0
3	1.0	1.2	1.6	0	.2	1
4	1.5	1.6	2.5	0	.1	1
5	2.1	2.5	2.9	0	.4	2
6	2.6	2.9	3.5*	0	.3	2
7	2.9				1.0 **	2
8	3.2					

* Total run time.

**Total waiting time.

Activity B begins at 0, and it takes .5 minute to complete the first part. B is then idle for .3 minute until part 2 arrives from A at .8 minutes. Part 2 takes .4 minute, so the ending time is .8 + .4 = 1.2 minutes. By this time part 3 has been waiting. 2 minute because it became available at .8 + .2 = 1.0 minute, but work could not be begun on it until 1.2 minutes. However, before activity B is finished on part 3 at 1.6 minutes, part 4 has arrived (at 1.0 + .5 = 1.5 minutes) and so one part is waiting. We continue systematically in this manner through part 6, noting that when it is finished at time were 3.5 minutes, there are two parts waiting, for their availability times were 2.9 minutes and 3.2 minutes, respectively.

(b) The average length of the waiting line (that is, average inventory) ahead of B can be expressed in equation form as follows:

$$\text{Average inventory} = \frac{\text{Total waiting time}}{\text{Total run time}}$$

$$= \frac{1.0\text{ assembly minute}}{3.5\text{ minutes}}$$

$$= 0.29\text{ assembly}$$

(c) Average output per hour:

$$\frac{6\text{ unit}}{3.5\text{ minutes}}\left(\frac{60\text{ min}}{\text{hr}}\right) = 102.9\text{ units / hr}$$

Illustration 48:

The Tit-Fit Scientific Laboratories is engaged in producing different types of high class equipment for use in science laboratories. The company has two different assembly lines to produce its most popular product 'Pressure'. The processing time for each of the assembly lines is regarded as a random variable and is described by the following distributions.

Process Time (minutes)	Assembly A1	Assembly A2
10	0.10	0.20
11	0.15	0.40
12	0.40	0.20
13	0.25	0.15
14	0.10	0.05

Using the following random numbers, generate data on the process times for 15 units of the item and compute the expected process time for the product. For the purpose, read the numbers vertically taking the first two digits for the processing time on assembly A1 and the last two digits for processing time on assembly A2.

4134	8343	3602	7505	7428
7476	1183	9445	0089	3424
4943	1915	5415	0880	9309

In the first stage, we assign random number intervals to the processing times on each of the assemblies.

Answer:

Computation of Random Interval for Processing Time

Process time Minutes	**A1**			**A2**		
	P_i	ΣP_i	Range	P_i	ΣP_i	Range
10	0.10	0.10	0-9	0.20	0.20	0-19
11	0.15	0.25	10-24	0.10	0.60	20-59
12	0.40	0.65	25-64	0.20	0.80	60-79
13	0.25	0.90	65-89	0.15	.095	80-94
14	0.10	1.00	90-99	0.05	1.00	95-99

Simulated date for 15 units

	Random No.	**Process Time**	**Random No.**	**Process Time**	**Total**
1	41	12	34	11	23
2	74	13	76	12	25
3	49	12	43	11	23
4	83	13	43	11	24
5	11	11	83	13	24
6	11	11	83	13	24
7	36	12	02	10	22
8	94	14	45	11	25
9	54	12	15	10	22
10	75	13	05	10	23
11	00	10	89	13	23
12	08	10	80	13	23

13	74	13	28	11	24
14	34	12	24	11	23
15	93	14	09	10	24
		182		167	349

Average Process time for
A1 = 182/15 = 12.13 Minutes
A2 = 167/15 = 11.13 Minutes
For product = 349/15 = 23.27 Minutes
Expected process time for the product = 23.27 minutes (12 .13 + 11.13)

Illustration 49:

A businessman is considering taking over a certain new business. Based on past information and his own knowledge of the business, he works out the probability distribution of the monthly costs and sales revenues, as given here:

Cost (in Rs.)	Probability	Sales Revenue (Rs.)	Probability
17000	0.10	19000	0.10
18000	0.10	20000	0.10
19000	0.40	21000	0.20
20000	0.20	22000	0.40
21000	0.20	23000	0.15
		24000	0.05

Use the following sequences of random numbers to be used for estimating costs and revenues. Obtain the probability distribution of the monthly net revenue.

Sequence 1	82	84	28	82	36	92	73	91	63	29
	27	26	92	63	83	02	10	39	10	10
Sequence 2	39	72	38	29	71	83	19	72	92	59
	49	39	72	94	04	92	72	18	09	00

Answer:

Cost (Rs.)	Probability	Cumulative Probability	Random Range	Cost (Rs.)	Probability	Cumulative Probability	Random Range
17000	0.1	0.1	00-09	19000	0.1	0.1	00.09
18000	0.1	0.2	10-19	20000	0.1	0.2	10-19
19000	0.4	0.6	20-59	21000	0.2	0.4	20-39
20000	0.2	0.8	60-79	22000	0.4	0.8	40-79
21000	0.2	1.0	80-99	23000	0.15	0.95	80-94
				24000	0.05	1.00	95-99

Month	Random No. for Cost	Cost (Rs.)	Random No. for Sales	Cost (Rs.)	Monthly Net Revenue (Rs.)
1	82	21000	39	21000	-
2	84	21000	72	22000	1000
3	28	19000	38	21000	2000
4	82	21000	29	21000	-
5	36	19000	71	22000	3000
6	92	21000	83	23000	2000
7	73	20000	19	20000	-
8	91	21000	72	22000	1000
9	63	20000	92	23000	3000

10	29	19000	59	22000	3000
11	27	19000	49	22000	3000
12	26	19000	39	21000	2000
13	92	21000	72	22000	1000
14	63	20000	94	23000	3000
15	83	21000	04	19000	(2000)
16	02	17000	92	23000	6000
17	10	18000	72	22000	4000
18	39	19000	18	20000	1000
19	10	18000	09	19000	1000
20	10	18000	00	19000	1000
					35000

Average = 35000/20 = Rs. 1750.

Illustration 50:

Table shows the time remaining (number of days until due date) and the work remaining (number of day's still required to finish the work) for 5 jobs which were assigned the letters A to E as they arrived to the shop. Sequence these jobs by priority rules viz., (a) FCFS, (b) EDD, (c) LS, (d) SPT and (e) LPT.

Job	Number days until due date	Number of day's of work remaining
A	8	7
B	3	4
C	7	5
D	9	2
E	6	6

Answer:

(a) FCFS (First come first served): Since the jobs are assigned letters A to E as they arrived to the shop, the sequence according to FCFS priority rule is A B C D E

(b) EDD (Early due date job first) rule: Taking into account the number of days until due date, the sequence of jobs as per EDD rules is

Job	B	E	C	A	D
No. of days units/due date	3	6	7	8	9

Here the job having earliest due date is sequenced first and the others are sequenced in ascending order of due date.

(c) L.S. (Least slack) rule also called as Minimum slack rule.

Calculation of slack:

Slack = (Number of days until due date) - (Number of days of work remaining)

Job	**No. of days until/due date**	**No. of days of work remaining**	**Slack (Days)**
A	8	7	8 – 7 = 1
B	3	4	3 – 4 = -1
C	7	5	7 – 5 = 2
D	9	2	9 – 2 = 7
E	6	6	6 – 6 = 0

Sequence:

Job	B	E	A	C	D
Slack -	1	0	1	2	7

Here the jobs are sequenced in ascending order of magnitude of their respective slacks.

(d) SPT (Shortest Processing Time job first) also referred as SOT (Shortest Operation time job First) rule or MINPRT (Minimum Processing time job first) rule. As per this rule, jobs are sequenced in ascending order of magnitude of their respective processing time.

Sequence:

Job	D	B	C	E	A
Processing Time (Days)	2	4	5	6	7

(e) LPT (Longest Processing time job first) also referred to as LOT (Longest operation time job first) rule.

As per this rule jobs are sequenced in desending order of magnitude of their respective processing times.

Sequence:

Job	A	E	C	B	D
Processing Time (Days)	7	6	5	4	2

Illustration 51:

The following jobs have to be shipped a week from now (week has 5 working days)

Job	A	B	C	D	E	F
Number of days of work remaining	2	4	7	6	5	3

Sequence the jobs according to priority established by (a) least slack rule (b) critical ratio rule.

Answer:

(a) Calculation of slack:

Number of days until due date is 1 week i.e. 5 days for all jobs

Job (1)	**No. of days until/due date (2)**	**No. of day of work remaining (3)**	**Slack (Days) (4) = (2) – (3)**
A	5	2	3
B	5	4	1
C	5	7	– 2
D	5	6	– 1
E	5	5	0
F	5	3	2

Sequence:

Job	C	D	E	B	F	A
Slack (Days) -	2	-1	0	1	2	3

Jobs are sequenced in ascending order of magnitude of respecive slack values.

(b) Calculation of Critical ratio:

$$= \frac{\text{Due date . Date Now}}{\text{Lead Time Remaining}} = \frac{\text{DD. DN}}{\text{LTR}} = \frac{\text{Available time till due date}}{\text{Operation time still needed to complete the job}}$$

Critical ratio for job A = 5/2 = 2.5

Critical ratio for job B = 5/4 = 1.25

Critical ratio for Job C = 5/7 = 0.71

Critical ratio for job D = 5/6 = 0.83

Critical ratio for job E = 5/5 = 1.0

Critical ratio for job F = 5/3 = 1.67

Job having least critical ratio is given the first priority and so on.

Sequence:	C	D	E	B	F	A
Critical Ratio:	0.71	0.83	1.0	1.25	1.67	2.5

Illustration 52:

In a factory, there are six jobs to perform, each of which should go through two machines A and B, in the order AB. The processing timings (in hours) for the jobs are given here. You are required to determine the sequence for performing the jobs that would minimise the total elapsed time, T. What is the value of T?

Job	Machine A	Machine B
1	7	3
2	4	8
3	2	6
4	5	6
5	9	4
6	8	1

Answer:

(a) The least of all the times given in the table is for job 6 on machine B. So, perform job 6 in the end. It is last in the sequence. Now delete this job from the given data.

(b) Of all timings now, the minimum is for job 3 on machine A. So, do the job 3 first.

(c) After deleting job 3 also, the smallest time of 3 hours is for job 1 on machine B. Thus, perform job 1 in the end (before job 6).

(d) Having assigned job 1, we observe that the smallest value of 4 hours is shared by job 2 on machine A and job 5 on machine B. So, perform job 2 first and job 5 in the end.

(e) Now, the only job remaining is job 4, it shall be assigned the only place left in the sequence. The resultant sequence of jobs is, therefore, as follows:

3	2	4	5	1	6

This sequence is the optimal one. The total elapsed time, T, is obtained in Table 2.8.16 as equal to 36 hours

Table: Calculation of Total Elapsed Time (T)

Job	**Machine A**		**Machine B**	
	In	**Out**	**In**	**Out**
3	0	2	2	8
2	2	6	8	16
4	6	11	16	22
5	11	20	22	26
1	20	27	27	30
6	27	35	35	36

As shown in this table, the first job, job 3, starts at lime 0 on the machine A and is over by time 2, when it passes to machine B to be worked on till time 8. The job 2 starts on the machine A at time 2 as the machine is free at that lime. It is completed at time 6 and has to wait for 2 hours before it is processed on machine B, starting at time 8 when this machine is free, Similarly, the various jobs are assigned to the two machines and the in and out times are obtained.

5. PRODUCTIVITY MANAGEMENT AND QUALITY MANAGEMENT

Questions For Classroom Discussion

Question 1 **(Illustration 1)**

In a particular plant there are 10 workers manufacturing a single product and the output per month consisting of 25 days of that particular product is 200. How much is the monthly productivity? (Monthly productivity per worker)

Note:

After this question, you should be able to complete Question No. 11 on your own. This question appeared in a previous online exam and was also tested in December 2024 for two marks.

Question 2 **(Illustration 11) Home Work**

A cement factory in Madhya Pradesh works 7 days a week in 3 shifts per days having maintenance in the first shift of around 2 hours. It has roughly 100 workers which produces only pozzolanic properties cement better known as PPC.

The output per month is around 2500 tonnes of PPC. Find the productivity per worker

Question 3 **(Illustration 2) Question also Appeared In A Previous Online Exam**

There are two industries A and B manufacturing hose couplings. The standard time per piece is 15 minutes.

The output of two small scale industries is 30 and 20 units respectively per shift of 8 hours.

Find the productivity of each per shift of 8 hours?

What is the expected production of each per week consisting of 6 days?

After this question, Students should be able to complete Question No. 12 on your own.

Question 4 **(Illustration 12) Home Work**

Compare the productivity of two plant of tobacco company situated in two different state Y and Z in an 8-hour shift. The standard time in manufacture a tobacco packet is 10 min. The output is 40 and 55 of two different plants in a shift.

Find also the expected productivity of both plants in a week.

Question 5 **Illustration 3**

The following data is available for a machine in a manufacturing unit:

Hours Worked	8
Working Days Per Month	25
Number of Operator	1
Standard Minutes Per Unit	
Machine Time	22
Operator Time	8
Total Time	**30**

Q.1 If plant is operated at 75% efficiency, and the operator is working at 100% efficiency, what is the output per month?

Q.2 If machine productivity is increased by 10% over the existing level, what will be the output per month?

Q.3 If operator efficiency is reduced by 20% over the existing level, what will be the output per month?

Note:

Question Number 3 has been tested in the main exam multiple times, with variations in the figures. It has been assigned 9 marks and 6 marks in different instances. Additionally, there is a mistake in the institute material solution to Question C.

Question 6 **(Illustration 13)**

For the given data of manufacturing unit which produces spare parts of HEMM the operators time, machine time and total time are 10, 28 and 38 minutes respectively. If there are one operator and working hour per day is 8 hour and considering 22 working days in a month. Find

(a) If plant is working at 65% efficiency, what is the expected output per month?
(b) If plant productivity is increased by 20% over the existing level, what will be the output per month?
(c) If operator efficiency is reduced by 30% due to injury over the existing level, what will be the output per month?

Please note that there is an error in the solution to Question C in the institute study material.

Question 7 **(Illustration 4)**

The following data is available for a manufacturing unit :
Number of operators =15
Daily working hours = 8
No. of days per month =25
Standard production per month =300 units
Standard Labour hours per unit = 8

The following information was obtained for November 2015:
Man days lost due to absenteeism = 30
Unit produced = 240
Idle Time = 276-man hours

Find the following
Q.1 Percent absenteeism
Q.2 Efficiency of utilization of labour
Q.3 Productive efficiency of labour
Q.4 Overall productivity of labour in terms of units produced per man per month.

After the fourth question, students can attempt Question 14. This question was tested in a previous year's online exam.

Question 8 **(Illustration 14) Home Work**

Following are the data related to call centre Firm which gives tech and non-tech support to large IT companies. It has 20 executives to address the queries which has 8 hour a shift having on an average 24 working days in month. On an average the company is able to address around 290 calls in a month.

Additional data for the current month is obtained
(a) No of call logged for the month = 250
(b) Idle time = 275-man hours
(c) Man days lost (absenteeism) = 28.

Question 9 **(illustration 5)**

An incentive scheme allows proportionate production bonus beyond 100% performance level. Calculate the amount of

Q.1 Incentive bonus and
Q.2 Total payment received by an operator on a particular day during which the following particulars apply:

Additional Information

✓ Operation - Assembling pocket transistor radio set

- ✓ Work Content - 30 Standard minutes per assembled set
- ✓ Attended Time : 8 Hours
- ✓ Time spent on unmeasured work : 2 Hours
- ✓ Numbers of sets assembled during the day : 15
- ✓ Wage rate : 4 per hour

Q.3 What is the net labour productivity achieved by the operator during the day?

After this question, students can attempt Question No. 15.

Question 10 **(Illustration 15) Home Work**

Compute the productivity per machine hour with the following data.

Also draw your interpretation.

Month	Number of Machine Employed	Working Hours	Production Units
January	400	220	99,000
February	550	180	100,000
March	580	220	125,000

Question 11 **(Illustration 6)**

Find the productivity of IT firm in terms of business achieved for the following data and comment

Quarter	No of Employees	Working Hours	Business Achieved
Q1	1,600	800	1,000,000
Q2	1,500	750	1,024,000
Q3	1,700	775	1,300,000
Q4	2,000	900	1,200,000

Question 12 **(Illustration 7)**

Calculate the standard production per shift of 8 hours duration, with the following data:

Observed time per unit = 5 minutes,

Rating Factor = 120%,

Total allowances = 30% of normal time.

Question 13 **(Illustration 16)**

Find the standard production for 8 hour shift. If allowance = 25% of normal time, Observer time per unit is 7 min and the rating factor is 110%.

Question 14 **(Illustration 8)**

Study in the packaging department of a soft drinks manufacturing unit revealed the following facts for a worker Basant Rao Patil.

Activity Element	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Performance Rating
Get Empty Cartoon (A)	0.15 minutes	0.25 minutes	****	0.17 minutes	90%
Place 30 Bottles in the Cartoon (B)	1.56 minutes	****	1.80 minutes	1.75 minutes	105%
Close the Cartton and Set Aside (C)	0.20 Minutes	****	0.10 minutes	0.15 minutes	95%

Smoking (D)	****	0.50 minutes	****	****	****

Calculate the standard production by Basant Rao in a shift of 8 hours when the units standard rules allow 10% as Allowance Factor.

Question 15 **(Illustration 9)**

A department works on 8 hours shift, 288 days a year and has the usage data of a machine, as given below:

Product	Annual Demand	Processing Time
A	325	5
B	450	4
C	550	6

Calculate

Q1) Processing time needed in hours to produce products A, B and C

Q2) Annual production capacity of one machine in standard hours, and

Q3) Number of machines required.

Question 16 **(Illustration 17)**

Find the standard production for 8 hr shift. If allowance = 25% of normal time, Observer time per unit is 7 min and the rating factor is 110%.

Answer:

Normal time per unit = Observed time / unit × Rating factor = 7 × (110/100) = 7.7 minutes

Now, Allowances = 25% of normal time = (25 × 7.7)/100 = 1.925 minutes.

So, Standard time/unit = Normal time/unit + Allowances = 7.7 + 1.925 = 9.625 minutes / unit

Hence, Standard production in shift of 8 hours = (8×60)/9.625 = 49.89 units (50 Units approx.)

Question 17 **(Illustration 10)**

Following results are recorded in a study of work sampling carried for 100 hours in a Machine Shop.

- ✓ Total number of observations recorded = 2500
- ✓ Number of observations in which no working activity is noticed = 400
- ✓ Ratio of Manual to Machine elements = 2:1
- ✓ Average Rating Factor = 115%
- ✓ Number of articles produced during the study period = 6000
- ✓ As per the policy of the company,
- ✓ Rest and personal allowances are taken as 12% of Normal Time.

1) Calculate Standard Time to produce an article?
Given that the shop produces 42000 articles per month of 25 working days by 5 workers working for a shift of 8 hours per day. Consider absenteeism to be 7%.

2) Compute Efficiency of utilization of Labour and

3) Productive Efficiency of Labour?

Question 18 **(Illustration 18)**

Below data are collected related to work study for 150 hrs on a floor shop employing 7 labours having a shift of 8 hrs in a day.

- ✓ Number of observations documented in total = 3000
- ✓ Number of observations in which no working activity is observed = 500
- ✓ Manual to machine ratio = 3:2
- ✓ Average Rating factor = 120%
- ✓ Number of product produced during the period of study = 7000
- ✓ Company has its own policy regarding personal allowance which is pegged at 11% of normal standard time to produce a product.

- ✓ The floor shop produces 49000 products per month for 24 working days, it has an absenteeism of around 6%.

1) Calculate efficiency of utilization of Labour and
2) Productive Efficiency of Labour.

5.1 Measurement Techniques of Productivity Index

Illustrations From Study Material

Illustration 1:

In a particular plant there are 10 workers manufacturing a single product and the output per month consisting of 25 days of that particular product is 200. How much is the monthly productivity?

Answer:

Monthly productivity per worker = 200 / 10 = 20 units

Illustration 2:

There are two industries A and B manufacturing hose couplings. The standard time per piece is 15 minutes. The output of two small scale industries is 30 and 20 respectively per shift of 8 hours. Find the productivity of each per shift of 8 hours. What is the expected production of each per week consisting of 6 days?

Answer:

$$\text{Productivity} = \frac{\text{Actual production}}{\text{Standard production}}$$

Standard production of hose complings per shift = $\frac{8 \times 60}{15}$ = 32 pcs.

Productivity of industry A = $\frac{30}{32} = \frac{15}{16}$ and productivity of industry B = $\frac{20}{32} = \frac{5}{8}$

If the productivity is expressed in percentage, the same for A is $\frac{15}{16} \times 100 = 93.75\%$

and productivity of industry B is $\frac{5}{8} \times 100 = 62.5\%$

Production per week of industry A = 30 × 6 = 180 nos. (Assuming the industry to work for one shift per day)

Production per week of industry B = 20 × 6= 120 nos. (Assuming the industry to work for one shift per day)

Illustration 3:

The following data is available for a machine in a manufacturing unit:

Hours worked per day	8
Working days per month	25
Number of operators	1
Standard minutes per unit of production	
Machine time	22
Operator time	8
Total time per unit	30

(i) If plant is operated at 75% efficiency, and the operator is working at 100% efficiency, what is the output per month?

(ii) If machine productivity is increased by 10% over the existing level, what will be the output per month?

(iii) If operator efficiency is reduced by 20% over the existing level, what will be the output per month?

Answer:

(a) Hours worked per day = 8

Working days per month = 25

Hours worked per month = 25 × 8 = 200 hrs.

Machine time = 22 minutes

Operator time = 8 minutes

Total time per unit = 30 minutes = $\frac{1}{2}$ hr.

No. of units that can be produced/month/operator = $\frac{200}{1/2}$ = 400

As the no. of operator is 1, possible monthly production = 400 units. As the plant operates at 75% efficiency.

Monthly production = 400 × $\frac{75}{100}$ = 300 units.

(b) If machine productivity is increased by 10% i.e. Machine time = 22 × $\frac{100}{(100+10)}$

= 20 minutes.

Then, total time = 20 + 8 = 28 minutes

Monthly production = $\frac{400 \times 30}{28} \times \frac{75}{100}$ = 321 Units

(c) If operator efficiency reduced by 20% i.e.

Operator time = 8 × $\frac{(100 + 20)}{100}$ = 8 x 1.2 = 9.6 minutes

Total time = 22 + 9.6 = 31.6 minutes.

Monthly production = $\frac{400 \times 30}{31.6} \times \frac{75}{100}$ = 284 Units

(Efficiency reduced by 20%. Instead of 100%, now 80% job is completed in 8 minutes. That means, operators time is increased to 10 minutes)

Illustration 4:

The following data is available for a manufacturing unit :

No. of operators	:	15
Daily working hours	:	8
No. of days per month	:	25
Std. production per month	:	300 units
Std. Labour hours per unit	:	8

The following information was obtained for November 2015:

Man days lost due to absentism	:	30
Unit produced	:	240
Idle Time	:	276 man hours

Find the following:—

(a) Percent absentism

(b) Efficiency of utilisation of labour

(c) Productive efficiency of labour

(d) Overall productivity of labour in terms of units produced per man per month.

Answer:

No. of days per month = 25

Daily working hrs. = 8

No. of operators = 15

No. of Man days per month = 15 × 25 = 375 Man days.

Total working hrs. per month = 375 × 8 = 3,000

Hours lost in absentism in a month = 30 × 8 = 240

(a) Percent absentism = $\frac{240 \text{ hrs.} \times 100}{3000 \text{ hrs}}$ = 8%

(b) Efficiency of utilisation of labour = $\frac{\text{Standard labour hour to produce 240 units}}{\text{Total Labour hour}} \times 100$

= $\frac{240 \times 8}{3000} \times 100$ = 64%

(c) Standard time required to produce 240units = 240 × 8 = 1920 labour-hours.

In November, man hours lost = 30 × 8 = 240

idle time (in hours) = 276

Total loss of time = 516 hours.

Productive hours available in November = 3000

Less, Total loss of time = (516)

Actual labour-hours = 2484 hours

Efficiency of labour $= \dfrac{\text{Std. Labour hrs}}{\text{Actual Labour hrs.}} = \dfrac{1920 \times 100}{2484} = 77.3\%$

(d) 15 men produces 300 units,

Std. labour productivity = 300/15 = 20 units.

In November, overall productivity = 240/15 = 16 units. (Ans.)

i.e. productivity falls by 25%.

Illustration 5:

An incentive scheme allows proportionate production bonus beyond 100% performance level. Calculate the amount of

(i) Incentive bonus and

(ii) Total payment received by an operator on a particular day during which the following particulars apply:

Operation : Assembling pocket transistor radio set
Work Content : 30 Standard minutes per assembled set
Attended Time : 8 Hours
Time spent on unmeasured work : 2 Hours
Numbers of sets assembled during the day : 15
Wage rate : Rs. 4 per hour

(iii) What is the net labour productivity achieved by the operator during the day?

Answer:

Total standard minutes worked during the day = 30 × 15 = 450, working time = 8 – 2 = 6 hours = 360 minutes.

Performance = (450 × 100) /360 = 125% i.e incentive is payable on 25% which is above 100%

(i) Incentive bonus = 0.25 × 6 × 4 = Rs. 6 for six hours on measured work

(ii) Guaranteed wage for 8 hours = 8 × 4 = Rs. 32; Total earnings for the days

= Rs. (6 + 32) = Rs. 38

(iii) Net labour productivity = Output in units / Net man hours = 15 / 6 = 2.5 sets per hour.

Illustrations From Study Material

Illustration 6:

Compute the productivity per machine hour with the following data. Also draw your interpretation.

Month	No. of machines employed	Working hours	Production Units
January	400	220	99,000
February	550	180	1,00,000
March	580	220	1,25,000

Answer:

Month	No. of machines employed	Working hours	Machine hours	Production Units
January	400	220	88,000	99,000
February	550	180	99,000	1,00,000
March	580	220	1,27,600	1,25,000

P = Productivity per machine hour
= Number of units produced /Machine hours
For January P = 99,000/88,000 = 1.125
February P = 100,000 / 99,000 = 1.010
March P = 125,000 /127,600 = 0.980

Interpretation: Though the total production in number of units is increasing, the productivity is declining.

Illustration 7:

Calculate the standard production per shift of 8 hours duration, with the following data: Observed time per unit = 5 minutes, Rating Factor -120%, Total allowances = 30% of normal time.

Answer:

Normal time per unit = Observed time / unit × Rating factor = 5 × (120/100) = 6minutes
Allowances = 30% of normal time = (30 × 6)/100 = 1.8 minutes
Standard time/unit = Normal time/unit + Allowances = 6 + 1.8 = 7.8 minutes / unit
Standard production in shift of 8 hours = (8 × 60)/7.8 = 61.54 units.

Illustration 8:

Study in the Packaging Department of a Softdrinks Manufacturing unit revealed the following facts for a worker Basant Rao Patil.

Cycle No.	**1**	**2**	**3**	**4**	**Performance Rating**
Activity Element					
(A) Get empty cartoon	0.15 min	0.25 min	—	0.17 min	90%
(B) Place 30 bottles in the cartoon	1.56 min	*	1.80 min	1.75 min	105%
(C) Close the cartoon & set aside	0.20 min	†	0.10 min	0.15 min	95%
(D) Smoking	—	0.50 min	—	—	—

* Bottles slipped out of hands and broke
† Empty cartoon not set aside and used for packaging in the next cycle.

Calculate the standard production by Basant Rao in a shift of 8 hours when the units standard rules allow 10% as Allowance Factor

Answer:

Average time for Activity Element A = $\frac{0.15 + 0.25 + 0.17}{3}$ = 0.19 min.

Average time for Activity Element B = $\frac{1.56 + 1.80 + 1.75}{3}$ = 1.703 min.

Average time for Activity Element C = $\frac{0.20 + 0.10 + 0.15}{3}$ = 0.15 min.

Computation of Normal Time

Activity Element	Average time (Mins)	Performance Rating (%)	Normal Time (Mins)	So Normal Time for the job of packaging = 2.101 Mins
(1)	(2)	(3)	(4) = (2) × (3) ÷ 100	
A	0.19	90	0.171	
B	1.703	105	1.788	
C	0.15	95	0.142	
Total	—	—	2.101	

Standard Time = $\frac{\text{Normal Time}}{1 - (\text{Allowance Factor} / 100)} = \frac{2.101}{1 - \frac{10}{100}}$ = 2.334 Mins

Standard Production in a shift of 8 hours = $\frac{8 \times 60}{2.334}$ = 205.66 cartoons.

Illustration 9:

A department works on 8 hours shift, 288 days a year and has the usage data of a machine, as given below:

Product	Annual Demand (units)	Processing time (Standard time in hours)
A	325	5.0
B	450	4.0
C	550	6.0

Calculate (a) Processing time needed in hours to produce products A, B and C, (b) Annual production capacity of one machine in standard hours, and (c) Number of machines required

Answer:

(a) The processing time needed in hours to produce products A, B and C in the quantities demanded using the standard time data:

Product	Annual Demand (units)	Processing time (standard time in hours)	Processing time needed to produce demand quantity (hrs.)
A	325	5.0	325 × 5 = 1,625
B	450	4.0	450 × 4 = 1,800
C	550	6.0	550 × 6 = 3,300
			Total = 6,725 hrs.

(b) Annual production capacity of one machine in standard hours = 8 × 288 = 2,304 hours per year.

(c) Number of machines required = Work load per year / Production capacity per Machine = 6,725 / 2,304 = 2.90 machines = 3 machines.

Illustration 10:

Following results are recorded in a study of work sampling carried for 100 hours in a Machine Shop.

1. Total no. of observations recorded — 2500
2. No. of observations in which no working activity is noticed — 400
3. Ratio of Mannual to Machine elements — 2 : 1
4. Average Rating Factor — 115%
5. No. of articles produced during the study period — 6000

As per the policy of the company, rest and personal allowances are taken as 12% of Normal Time. Calculate Standard Time to produce an article.

Given that the shop produces 42000 articles per month of 25 working days by 5 workers working for a shift of 8 hours per day. Consider absentism to be 7%.

Compute Efficiency of utilisation of Labour and Productive Efficency of Labour.

Answer:

Percentage of working time = $\frac{2500 - 400}{2500} \times 100 = 84\%$

Actual working time in a study of 100 hours = 84 hours = 84 × 60 = 5040 mins.

Production — 6000 articles

Time required to produce an article = $\frac{5040}{6000} = 0.84$ mins

Of this Manual time = 0 84 x 2/ 3 ($\therefore$ Ratio of Manual to Machine activity elements = 2:1)

= 0.56 mins

Machine time = $0.84 \times \frac{1}{3} = 0.28$ min.

Normal Time of man = Time of man as per study × Rating Factor / 100

= $0\ 56 \times \frac{115}{100} = 0.644$ min.

Normal Time of machine = 0.28 min.

Allowances for man = 12% of Normal time of Man = 0.12 × 0.644 = 0.077 min

Standard Time for Man to produce an article = Normal Time of Man + Allowances

= 0.644 + 0.077 = 0.721 min.

Standard Time for machine = 0.28 min.

Standard Time to produce an article = 0.28 + 0.721 = 1.001 mins.

Standard time required to produce 42000 articles = 42000 × 1.001 = 42042 mins. = 700.7 hours.

No. of days/month - 25, Daily working hours - 8, No. of workers - 5

Total available working hours/month = 5 × 25 × 8 = 1000

Actual working hours/month = 1000 × 0.93 [Since Absentism = 7%]

= 930

Efficiency of utilisation of Labour $= \frac{\text{Standard time to produce 42000 articles}}{\text{Total available hours}} \times 100$

$= \frac{700.7}{1000} \times 100 = 70.07\ \%$

Productive Efficiency of Labour$= \frac{\text{Standard time to produce 42000 articles}}{\text{Actual Working hours}} \times 100$

$= \frac{700.7}{930} \times 100 = 75.34\%$

Illustration 11:

A cement factory in Madhya Pradesh works 7 days a week in 3 shifts per days having maintenance in the first shift of around 2 hours. It has roughly 100 workers which produces only pozzolanic properties cement better known as PPC. The output per month is around 2500 tonnes of PPC. Find the productivity per worker?

Answer:

Productivity per worker = 2500/100 = 25 tonnes.

Illustration 12:

Compare the productivity of two plant of tobacco company situated in two different state Y and Z in an 8-hour shift. The standard time in manufacture a tobacco packet is 10 min. The output is 40 and 55 of two different plants in a shift. Find also the expected productivity of both plants in a week.

Answer:

Productivity =Actual production/Standard production
Standard production of tobacco plant is = 8 × 60 / 10 = 48 packets.
Productivity of plant located in state Y = 40/48 = 0.833 (83.33%)
Productivity of plant located in state Z = 55/48 = 1.146 (114.6%)
Now, expected productivity of plant in Y = 40 × 7 = 280 Packets (if it works for 7 days with one shift)
And, expected productivity of plant in Z = 55 × 7 = 385 Packets (if it works for 7 days with one shift)

Illustration 13:

For the given data of manufacturing unit which produces spare parts of HEMM the operators time, machine time and total time are 10, 28 and 38 minutes respectively. If there are one operator and working hour per day is 8 hr and considering 22 working days in a month. Find

(a) If plant is working at 65% efficiency, what is the expected output per month?
(b) If plant productivity is increased by 20% over the existing level, what will be the output per month?
(c) If operator efficiency is reduced by 30% due to injury over the existing level, what will be the output per month?

Answer:

Working hours per month = 22 × 8 = 176 hrs.
No. of units that can be produced/month by the operator = 176 × 60/38= 277.89 approx 278.

a. Now if the plant efficiency = 65% and since there is only one operator and its efficiency is 100% then expected production of spare parts = 277.89 × 0.65 = 180.62 Units.
b. If the plant efficiency increases by 20% new output will be
   New machine time is 28 × 100/120 = 23.33 minutes, and then the total time = 10 + 23.33 = 33.33 min
   New monthly production = 277.89 × (38/33.33) × 0.65 = 205.93 Units
c. If the operator's efficiency is reduced by 30% then new production will be
   New Operator's time = 10 × (130/100) = 13 min
   So new total time = 13 + 28 = 41 minutes.
   Now new monthly production = 277.89 × (38/41) × 0.65 = 167.41 units

Illustration 14:

Following are the data related to call centre Firm which gives tech and non tech support to large IT companies.

It has 20 executives to address the queries which has 8 hr a shift having on an average 24 working days in month.

On an average the company is able to address around 290 calls in a month.

Additional data for the current month is obtained

(a) No of call logged for the month = 250
(b) Idle time = 275-man hours
(c) Man days lost (absenteeism) = 28.

Find

1. Efficiency of utilisation of manpower
2. Absenteeism (%)
3. Overall productivity of manpower.

Answer:

No of mandays per month = 20 × 24 = 480 Man days
Total working hr per month = 480 × 8 = 3840.
Hr lost in absenteeism in a month = 28 × 8 = 224

1. Efficiency of utilisation of manpower = (250 × 8 / 3840) × 100 = 52.08%
2. Absenteeism = (224 hr/3840hr) ×100 = 5.833%.

3. 20 men logs 290 calls in a month

So the St. manpower productivity = 290/20 = 14.5 calls.

In the current month overall productivity = 250/20 = 12.5 calls

So, the productivity has fallen from 14.5 to 12.5 i.e. 13.8%

Illustration 15:

Find the productivity of IT firm in terms of business achieved for the following data and comment

Quarter	No of Employees	Working Hours	Business Achieved (Rs.)
Q1	1600	800	1000000
Q2	1500	750	1024000
Q3	1700	775	1300000
Q4	2000	900	1200000

Answer:

Quarter	**No of Employees**	**Working Hours**	**Man Hours**	**Business Achieved (Rs.)**	**Productivity**
Q1	1600	800	1280000	1000000	0.78125
Q2	1500	750	1125000	1024000	0.910222
Q3	1700	775	1317500	1300000	0.986717
Q4	2000	900	1800000	1200000	0.666667

Man hour of Q1 = No of Employee × Working Hours = 1600 × 800 = 1280000

Productivity in Q1 = 1000000/1280000 = 0.78125

From the above table we can say that the productivity of Q3 is best then follows Q2 then Q1 and the least is Q4.

Illustration 16:

Find the standard production for 8 hr shift. If allowance = 25% of normal time, Observer time per unit is 7 min and the rating factor is 110%.

Answer:

Normal time per unit = Observed time / unit × Rating factor = 7 × (110/100) = 7.7 minutes

Now, Allowances = 25% of normal time = (25 × 7.7)/100 = 1.925 minutes.

So, Standard time/unit = Normal time/unit + Allowances = 7.7 + 1.925 = 9.625 minutes / unit

Hence, Standard production in shift of 8 hours = (8×60)/9.625 = 49.89 units (50 Units approx.)

Illustration 17:

A captive plant works for one shift in a day i.e. 8 hr a shift for 200 days in a year to cater for large automobile company. It produces three product having annual demand as 425, 429 and 546 units respectively. The processing time (standard time in hr) are 4, 5 and 5.5 hours respectively. Calculate

(a) Processing time required to produce all three products.

(b) Annual production

(c) And number of machine required.

Answer:

(a) Product 1 → Processing time needed toproduce demand quantity (hrs.) = Annual Demand × Processing time (Standard time in hr) = 425 × 4 = 1700 hrs

Product 2 → Processing time needed to produce demand quantity (hrs.) = 429 × 5 = 2145 hrs

Product 3 → Processing time needed to produce demand quantity (hrs.) = 546 × 5.5 = 3003 hrs.

Hence total time needed= 6848hrs.

(b) Annual production capacity for a single machine = 8 × 200 =1600 hrs for a year.

(c) Minimum number of machines required = 6848/1600 = 4.29 (5 machine Approx.)

Illustration 18:

Below data are collected related to work study for 150 hrs on a floor shop employing 7 labours having a shift of 8 hrs in a day.

(a) Number of observations documented in total = 3000
(b) Number of observations in which no working activity is observed = 500
(c) Manual to machine ratio = 3:2
(d) Average Rating factor = 120%
(e) Number of product produced during the period of study = 7000

Company has its own policy regarding personal allowance which is pegged at 11% of normal standard time to produce a product.

The floor shop produces 49000 products per month for 24 working days, it has an absenteeism of around 6%.

Calculate efficiency of utilisation of Labour and Productive Efficiency of Labour.

Answer:

Percentage of working time = ((3000-500)/3000) × 100 = 83.33%
Actual working time in a study of 150 hrs = 150 × 0.8333 × 60 = 7500 min.
Production = 7000 units.
Time required for one unite to produce = 7500/7000 = 1.0714 min
So, manual time on this is 1.0714 × (3/5) = 0.643 mins and machine time is 1.0714 × (2/5) = 0.43 mins.
Normal time of labour = time of labour as per study × Rating Factor/100 = 0.643 × 120/100= 0.772 min.
And normal time of machine = 0.43 min.
Now, if allowance is considered which is 11% of normal time which bring the standard time for the labour to produce = 0.772 × 1.11 = 0.857 min.
Hence Standard time required to produce a product = 0.857 + 0.43 = 1.286 min.
And. Standard time required to produce 49000 units = 49000 × 1.286 = 63,038 min = 1050 hrs.
Now, total available working hrs for 24 working day of 8 hrs shift of 7 labours = 1344 hr in a month
Taking absenteeism in consideration actual working hour left = 1344 × 0.94 = 1263.4 hrs.
Efficiency of utilisation of Labour= 1050/1344 = 0.7813 (78.13%)
And
Productive efficiency of Labour = 1050/1263.4 = 0.8312 (83.12%)

6. Project Management,Monitoring and Control

Questions For Classroom Discussion

Question 1

Draw the network diagram to the following activities

Activity	Time Duration
1 to 2	2
1 to 3	4
1 to 4	3
2 to 5	1
3 to 5	6
4 to 6	5
5 to 6	7

Question 2

A project schedule has the following characteristics.

Activity	Duration
1 to 2	4
1 to 3	1
2 to 4	1
3 to 4	1
3 to 5	6
4 to 9	5
5 to 6	4
5 to 7	8
6 to 8	1
7 to 8	2
8 to 10	5
9 to 10	7

Q.1 Construct network diagram

Q.2 Compute earliest latest time for each event

Q.3 Find EST, LST, EFT, LFT values and Total Float of all activities

Q.4 Find critical path and project duration.

Question 3

A project has the following time schedule.

Q.1 Construct network diagram

Q.2 Compute EST, LST, EFT, LFT and Total Float of all activities.

Q.3 Find critical path and its duration.

Activity	Duration
1 to 2	2
1 to 3	2
1 to 4	1
2 to 5	4
3 to 6	8
3 to 7	5

4 to 6	3
5 to 8	1
6 to 9	5
7 to 8	4
8 to 9	3

Question 4

A project has the following time schedule.

Q.1 Construct network diagram

Q.2 Compute Earliest and Latest Event time

Q.3 Compute EST,LST,EFT,LFT and Total Float of all activities.

Question 5

Find critical path and its duration.

Activity	Duration
1 to 2	20
1 to 3	25
2 to 3	10
2 to 4	12
3 to 4	6
4 to 5	10

Program Evaluation and Review Technique (PERT)

Question 6

Assuming that the expected time are normally distributed.

Find critical path and project duration.

Activity	Optimistic	Most Likely	Pessimistic
1 to 2	2	5	14
1 to 3	9	12	15
2 to 4	5	14	17
3 to 4	2	5	8
3 to 5	8	17	20
4 to 5	6	9	12

Computation of Probability

Less Than	**Greater Than**
If Z is +ve Probability = 0.5 + Table Value	If Z is +ve Probability = 0.5 - Table Value
If Z is -ve Probability 0.5 - Table Value	If Z is -ve Probability = 0.5 + Table Value

Question 7

Mean and Standard Deviation of a project duration are 300 and 100 days respectively. Find the probability for

Q.1 Completing the project within 417 days

Q.2 Not completing within 417 days

Q.3 Completing the project within 183 days
Q.4 Not completing within 183 days.
(Area under normal distribution from z = 0 to z = 1.17 is 0.3790)

Standard Normal Distribution Table

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Question 8

A project is expected to take 15 months along the critical path having standard deviation of 3 months. What is the probability of completing the project

Q.1 Within 18 months

Q.2 Within 12 months

Area Under Standard Normal Curve from Z=0 to Z=1 is 0.3413

Question 9

Q.1 Draw Network Diagram

Q.2 Calculate length and variance of the critical path

Q.3 What is the approximate probability that jobs on the critical path will be completed within (a)41 days (b) 35 days.

Q.4 What is the probability that the project will not be completed within 45 days.

Area under normal distribution from z=0 to z=1 is 0.3413

Area under normal distribution from z=0 to z=0.2 is 0.0793

Area under normal distribution from z=0 to z=1.8 is 0.4641

Three Important Questions

Question 10

A project consists of five activities. Activities P and Q run simultaneously. The relationship among the various activities is as follows.

Activity	Immediate Successor
P	R
Q	S

Activity T is the last operation of the project and it is also immediate successor to R and S.

Draw the network of the project.

Question 11

A project work consists of seven activities. Activities P,Q,R run simultaneously. The relationship among various activities is as follows.

Activity	Immediate Successor
P	S
Q	T
R	U

Activity “v” is the last operation of the project and it is also immediate successor to S,T, and U.

Draw the network diagram.

Question 12

Project with the following data is to be implemented. Draw the network diagram and find the critical path.

Activity	Predecessor	Duration	Cost
A	-------	2	50
B	-------	4	50
C	A	1	40
D	B	2	100
E	A,B	3	100
F	E	2	60

Q.1 What is the minimum duration of the project?

Q.2 Draw a Gantt chart for early start schedule

Time cost Trade off Analysis- Crashing

The following table gives duration in days and cost of the activities for a project

Activity	Normal Time	Crash Time	Normal Cost	Crash Cost
1 to 2	4	3	600	800
1 to 3	2	2	400	400
1 to 4	5	4	750	900
2 to 3	7	5	400	600
2 to 5	7	6	800	1000
3 to 5	2	1	500	650
4 to 5	5	4	600	850
			4050	

Indirect cost per day is Rs 200

Q.1 Draw the network of the project

Q.2 Find normal duration and cost of the project

Q.3 Find the optimum duration and cost of the project.

Illustration From Study Material

Illustration 1:

A project consists of seven activities. Activities P, Q, R run simultaneously. The relationships among the various activities is as follows:

Activity	Immediate Successor
P	S
Q	T
R	U

Activity "V is the last operation of the project and it is also immediate successor to S, T and U. Draw the network of the project.

Answer:

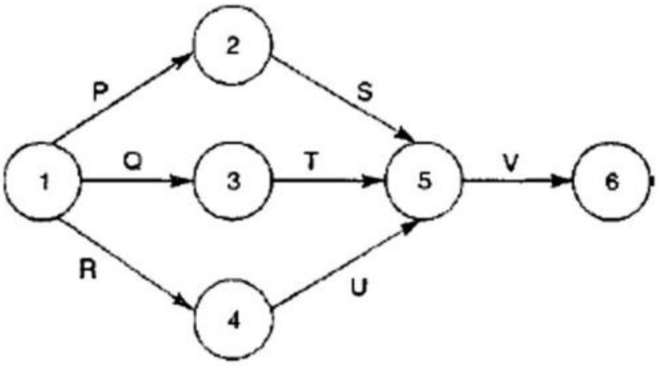


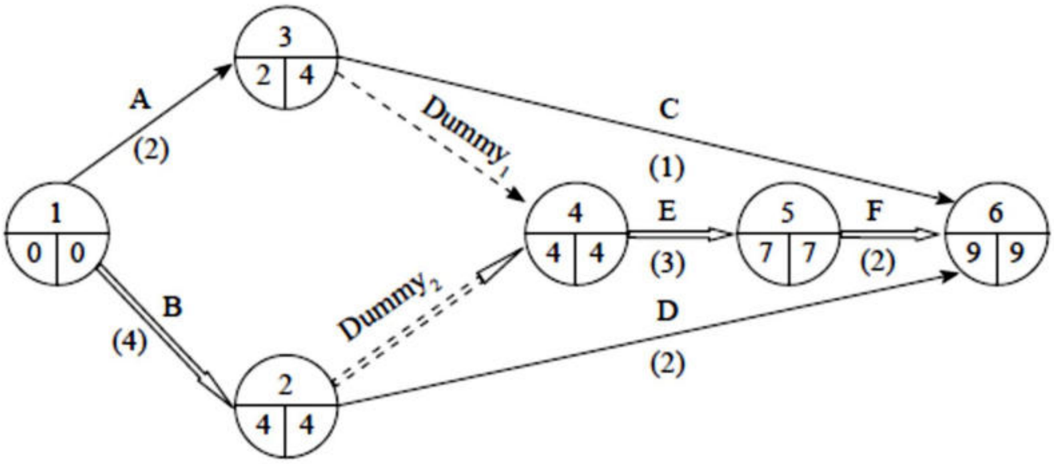
Illustration 2:

Project with the following data is to be implemented. Draw the network and find the critical path.

Activity	Predecessor	Duration (days)	Cost (Rs. Day)
A	-	2	50
B	-	4	50
C	A	1	40
D	B	2	100
E	A,B	3	100
F	E	2	60

1. What is the minimum duration of the project?
2. Draw a Gantt chart for early start schedule.
3. Determine the peak requirement of money and the day on which it occurs in the above schedule.

Answer:



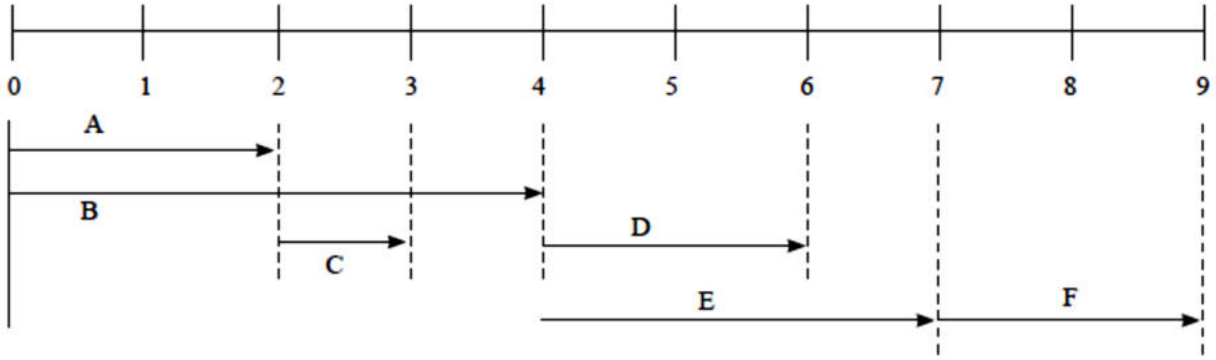
Critical Path B – Dummy2 – E – F

Minimum duration of the project = 9 days

Table: Activity Relationship

Activity	t	ES (EF-t)	EF	LS (LF-t)	LF	Event Slack (LS-ES) (LF-EF)	On Critical Path
A	2	0	2	2	4	2	No
B	4	0	4	0	4	0	Yes
C	1	4	5	8	9	4	No
D	2	4	6	7	9	3	No
E	3	4	7	4	7	0	Yes
F	2	7	9	7	9	0	Yes

(2) Gantt Chart for Early Start Schedule



(3) Peak requirement of money will occur during simultaneous occurance of Activities.

From the Network diagram above, it can be said that the following Activities need to occur simultaneously.

(i)	A & B	—	Either during the days 1 & 2 or during the days 3 & 4 of Project Duration, which will require (Rs. 50 for A + Rs. 50 for B) per day i.e. Rs. 100 per day
(ii)	B & C	—	Either on day 3 or on day 4 of the project and it will require (Rs. 50 for B + Rs. 40 for C) per day i.e. Rs. 90 per day
(iii)	C, D & E	—	During day no. 5 or day no. 6 and cost is Rs. (40 + 100 + 100) = Rs. 240 per day
(iv)	C, D & F	—	During day no. 8 or day no. 9 and cost is Rs. (40 + 100 + 60) = Rs. 200 per day
(v)	D & E	—	During day nos. 5 & 6 or 6 & 7. Cost is Rs. (100 + 100) = Rs. 200 per day
(vi)	D & F	—	During day nos. 8 & 9. Cost = Rs. (100 + 60) = Rs. 160 per day
(vii)	C & E	—	Either on day no. 5 or 6 or 7. Cost to be incurred = Rs. (40 + 100) = Rs. 140 per day

From above we can say that C can occur by using either of the options (ii), (iii), (iv) & (vii). As cost fo option (ii) is least one should decide fo it at a cost of Rs. 90 per day.

Similarly D can occur by either of the option (iii), (iv), (v) & (vi) above. As (vi) is the least cost option of all these, one should go for it at a cost of Rs. 160 per day.

Hence the Project Activities should follow the sequence given below:-

(a) A & B to start at their Earliest Time (i.e 0) and occur simultaneously till day 2 @ Rs. 100 per day

(b) C can start either at its Earliest Time (i.e. 2) or on day 3 and occur simultaneously with B either on day 3 or 4 @ Rs. 90 per day

(c) E being Critical Activities must have to start at it's earliest time (i.e. 4) and occur @ Rs. 100 per day

(d) F being Critical Activity has to start on Earliest Time (i.e. 7) and will occur concurrently with D is the during days 8 & 9 @ Rs. 160 per day.

Hence peak requirement of money is Rs. 160 per day and it will occur at days 8 and 9.

Illustration 3:

A project has the following time schedule

Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9	7-8	8-9
Time (months)	2	2	1	4	8	5	3	1	5	4	3

Construct a PERT network and compute

- Critical path and its duration
- Total float for each activity

Also, find the minimum number of cranes the project must have for its activities 2-5, 3-7, 5-8 and 8-9 without delaying the project given that one crane is sufficient to carry out the work involved in each activity if taken care of individually.

Answer:

Steps:

1. Moving forward, find EF times (choosing the Maximum at activity intersection)
2. Maximum EF = LF = Critical Path Time
3. Return path find LF (Choosing the Minimum at activity intersection)
4. Note LF, EF from network (except activity intersections)

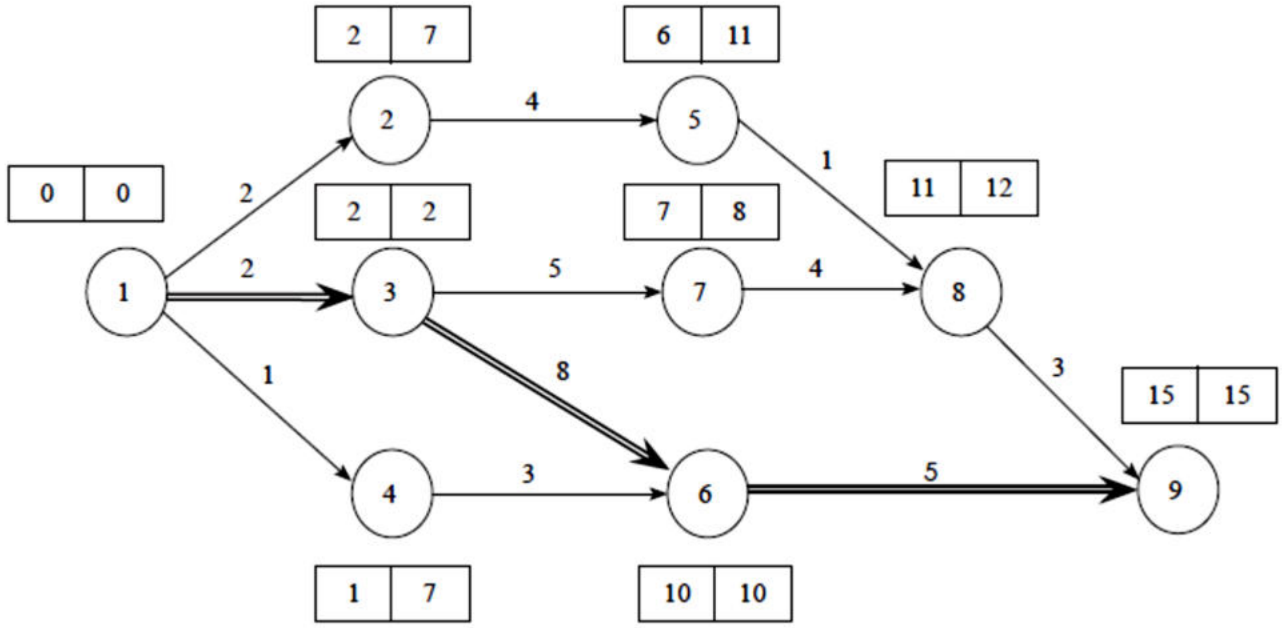


Table: Activity Relationship

Activity	Duration Months (tij)	Earliest Start (ESij)	Earliest Finish (EFij = ESij + tij)	Latest Start (LSij = LFij – tij)	Latest Finish (LFij)	Total Float (TFij = LSij + ESij = LEij – EFij)
1 -2	2	0	2	5	7	5
1 -3	2	0	2	0	2	0
1 -4	1	0	1	6	7	6
2-5	4	2	6	7	11	5
3-6	8	2	10	2	10	0
3-7	5	2	7	3	8	1
4-6	3	1	4	7	10	6
5-8	1	6	7	11	12	5
6-9	5	10	15	10	15	0
7-8	4	7	11	8	12	1

8-9	3	11	14	12	15	1

Critical path is 1-3-6-9 with duration 15 months

Minimum number of cranes

- Finish 3 — 7 at its earliest finish time 7 with one crane
- Finish 2 — 5 at its latest finish time 7 + 4 =11 with the same crane by starting the activity at its latest start time 7
- Finish 5 — 8 at its latest finish time 11 + 1 = 12 with the same crane by starting the activity at its latest start time 11
- Finish 8 — 9 at its latest finish time 12+ 3=15 with the same crane by starting the activity at its latest start time 12

Therefore, one crane will be sufficient if start time of the following activities are:

- Activities 2-5 — 7
- Activities 5-8 — 11
- Activities 8-9 — 12

Illustration 4:

A project consists of five activities. Activities P and Q run simultaneously. The relationship among the various activities is as follows:

Activity	Immediate Successor
P	R
Q	S

Activity T is the last operation of the project and it is also immediate successor to R and S. Draw the network of the Project.

Answer:

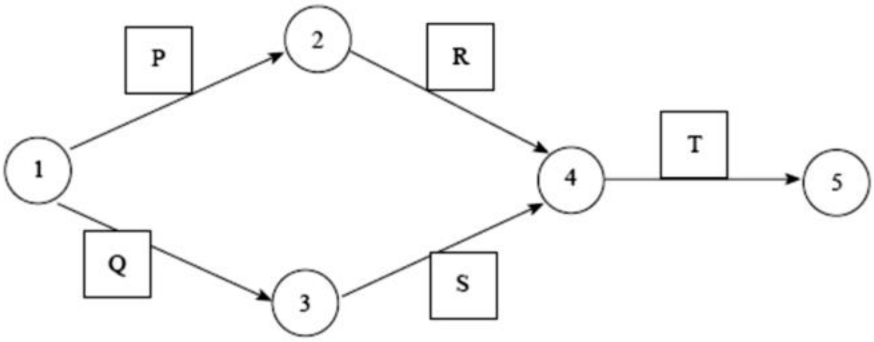


Illustration 5:

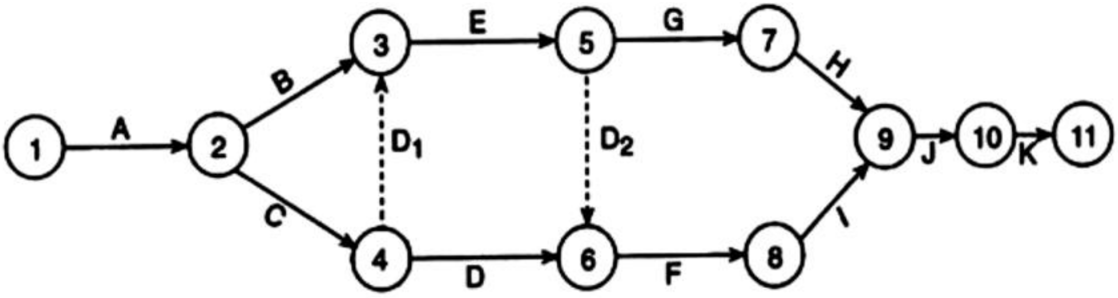
XYZ Auto-manufacturing company has to prepare a design of its latest model of motorcycle. The various activities to be performed to prepare design are as follows:

Activity	Description of activity	Preceding activity
A	Prepare drawing	—
B	Carry out cost analysis	A
C	Carry out financial analysis	A
D	Manufacture tools	C
E	Prepare bill of material	B, C
F	Receive material	D,E
G	Order sub-accessories	E
H	Receive sub-accessories	G
I	Manufacture components	F
J	Final assembly	I,H
K	Testing and shipment	J

Prepare an appropriate network diagram.

Answer:

The network diagram will be as follows:



Where D1 and D2 are dummy activities

Illustration 6:

The following table gives data on normal time & cost and crash time & cost for a project.

Activity	Normal		Crash	
	Time (days)	Cost (Rs.)	Time (days)	Cost (Rs.)
1—2	6	600	4	1,000
1—3	4	600	2	2,000
2—4	5	500	3	1,500
2—5	3	450	1	650
3—4	6	900	4	2,000
4—6	8	800	4	3,000
5—6	4	400	2	1,000
6—7	3	450	2	800

The indirect cost per day is Rs. 100.

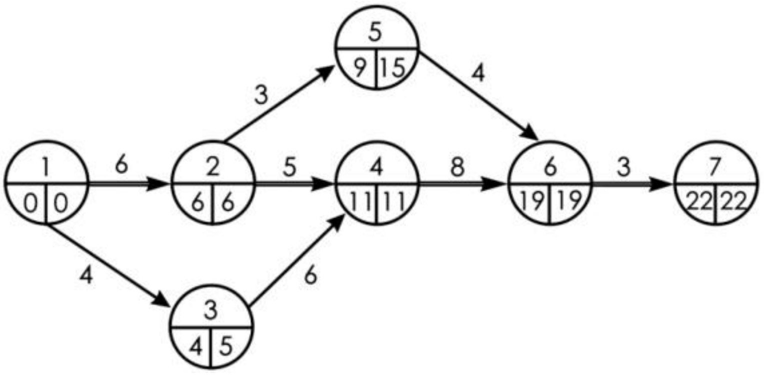
(i) Draw the network and identify the critical path.

(ii) What are the normal project duration and associated cost?

(iii) Crash the relevant activities systematically and determine the optimum project completion time and cost.

Answer:

(i) The network for normal activity times indicates a project time of 22 days with the critical path 1-2-4-6-7.



(ii) Normal project duration is 22 days and the associated cost is as follows:

Total cost = Direct normal cost + Indirect cost for 22 days.

= 4,700 + 100 × 22 = Rs. 6,900.

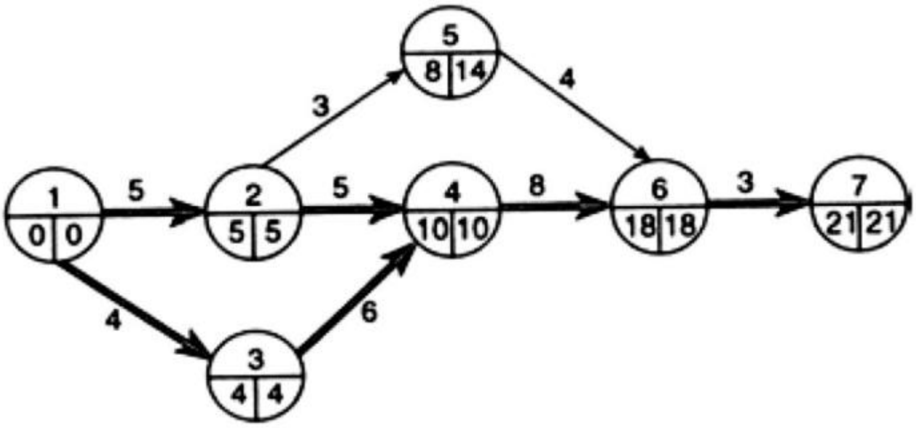
(iii) For critical activities, cost - slope is given below:

Critical activity	Cost-slope* (Rs./day)	*Cost slope= $\frac{\text{Crash Cost-Normal Cost}}{\text{Normal Time-Crash Time}}$
1-2	$\frac{1000 - 600}{6 - 4} = 200$	

2-4	$\frac{1500 - 500}{5 - 3} = 500$	
4-6	$\frac{3000 - 800}{8 - 4} = 550$	
6-7	$\frac{800 - 450}{3 - 2} = 350$	

Of the activities lying on the critical path, activity 1—2 has lowest cost slope Therefore, we shall first crash this activity by just one day.

Duration = 21 days, and cost = 4700 + 1 × 200 + 100 × 21 = Rs. 7000.



Other activities too have become critical. Now we have 2 critical paths:

1→2→4→6→7 and 1→3→4→6→7.

To reduce duration of the activity further, we shall have to reduce duration of both the paths. We have following alternatives:

Crash activity 6 — 7 by 1 day at a cost of Rs. 350.

Crash activity 4 — 6 by 4 days at the cost of Rs. 550 per day.

Crash activities 1—2 and 1 — 3 by 1 day each at a cost of Rs. (200 + 700) = Rs. 900.

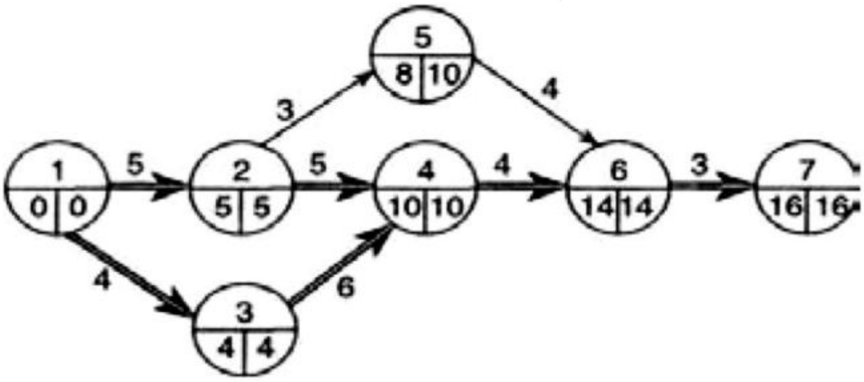
Crash activities 2 — 4 and 3 — 4 by 2 days each at a cost of Rs. (500 + 550) = Rs. 1050/day.

Thus, we shall first crash activities 6 — 7 by 1 day and then activity 4 — 6 by 4 days.

On crashing activity 6 — 7 by 1 day, cost = 4900 + 350 × 1 + 100 × 20 = Rs. 7250, and duration = 20 days.

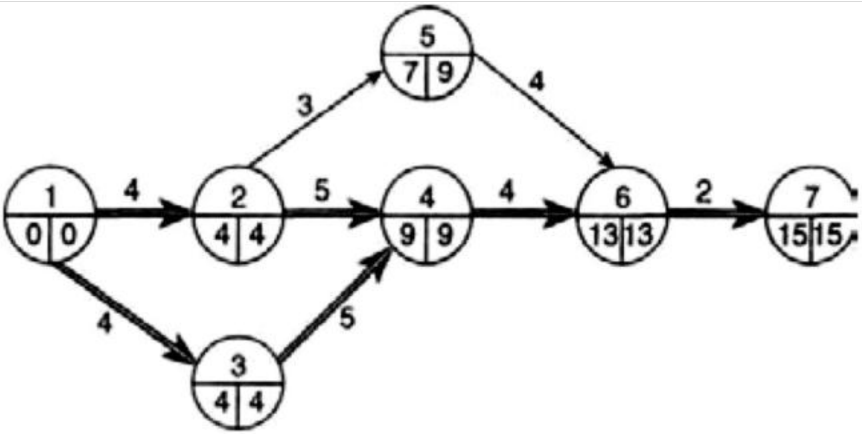
Next we crash 4—6 by 4 days.

Cost = 5250 + 550 × 4 + 100 × 16 = Rs. 9050. Duration = 16 days.

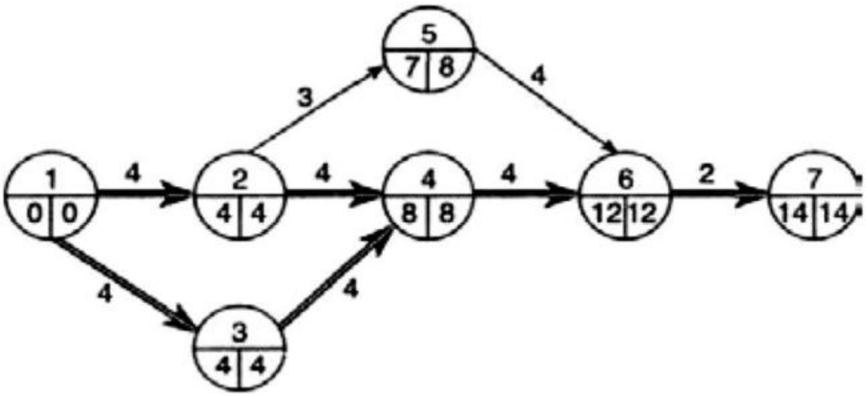


Next we crash activities 1—2 and 3—4 by 1 day each.

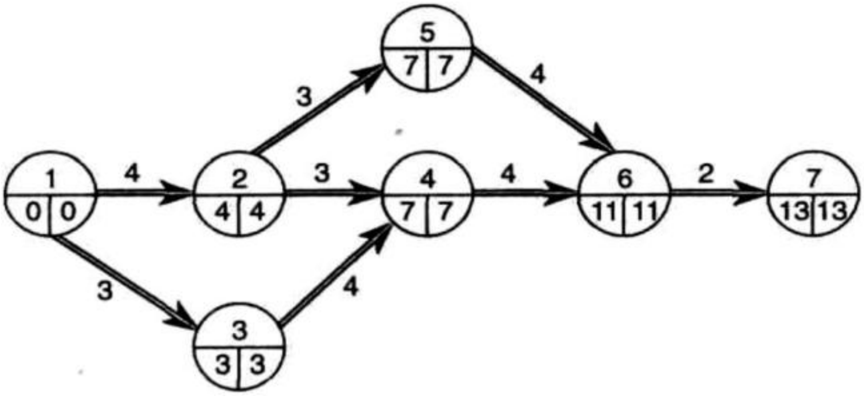
Cost = 7450 + 200 × 1 + 550 × 1 + 100 × 15 = Rs. 9700.



Next we crash activities 2—4 and 3—4 by 1 day each.
Cost = 8200 + 500 × 1 + 550 × 1 + 100 × 14 = Rs. 10,650. Duration = 14 days.



We crash activities 1—3 and 2—4 by 1 day each.
Cost = 9250 + 700 × 1 + 500 × 1 + 100 × 13 = Rs. 11,750 Duration = 13 days.



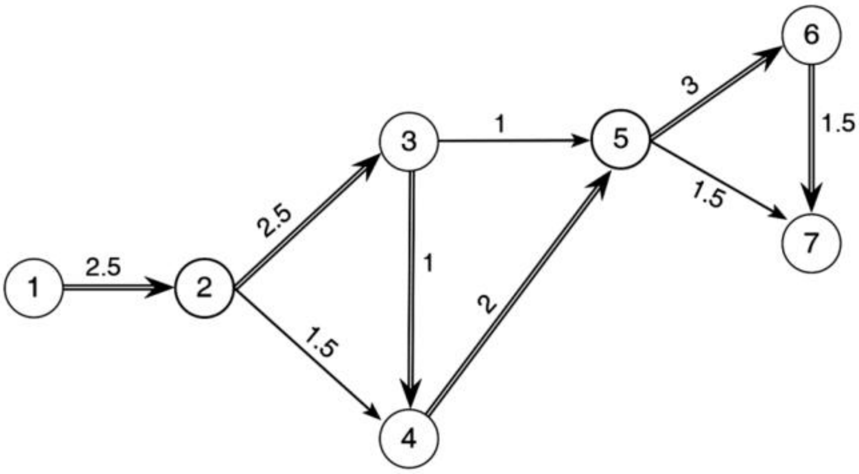
Now there are three critical paths:
1—2—5—6—7, 1—2—4—6—7, 1—3—4—6—7
Also, no further crashing is possible. Hence minimum duration of the project =13 days with cost Rs. 11,750

Illustration 7:

Draw the network for the following activities and find critical path and total duration of project.

Activity	Duration (months)	Activity	Duration (months)
1-2	2.5	4-5	2.0
2-3	2.5	5-6	3.0
2-4	1.5	6-7	1.5
3-4	1.0	5-7	1.5
3-5	1.0		

Answer:



Paths	Duration
1-2-3-5-6-7	2.5+2.5+1+3+1.5 = 10.5
1-2-3-5-7	2.5+2.5+1+1.5 = 7.50
1-2-3-4-5-6-7	2.5+2.5+1+2+3+1.5 = 12.5 (Critical path)
1-2-3-4-5-7	2.5+2.5+1+2+1.5 = 9.5
1-2-4-5-7	2.5+1.5+2+1.5 = 7.5
1-2-4-5-6-7	2.5+1.5+2+3+1.5 = 10.5

Illustration 8:

The following activities must be accomplished in order to complete a construction project:

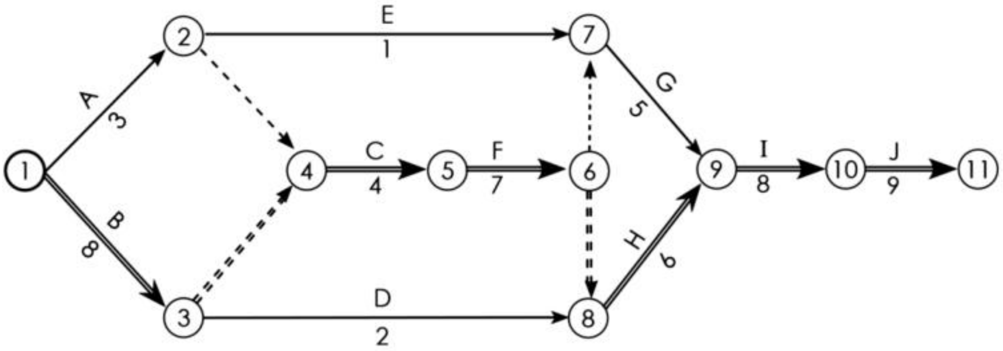
Activity	A	B	C	D	E	F	G	H	I	J
Time	3	8	4	2	1	7	5	6	8	9
Predecessors	—	—	AB	B	A	C	EF	DF	GH	I

- Construct a network diagram for this project. Find the CP and the duration of the project.
- Assume that you are project manager of the project mentioned above. The project has progressed for 10 weeks and the status is follows:

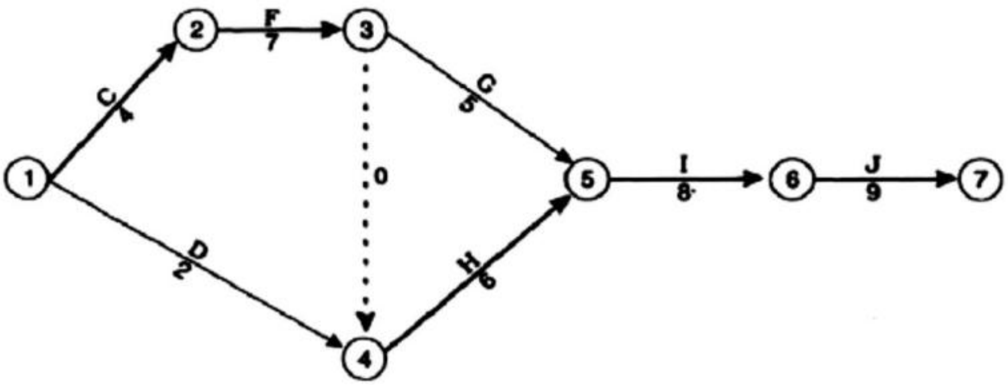
Activities completed: A, B, E. Other activities have not started as yet.

- If no managerial action is taken at all when will the project get completed?
- What action might you take to get the project back to a schedule that can be completed by the end of week 42?

Answer:



Paths	Duration (weeks)	Paths	Duration (weeks)
1-2-7-9-10-11	26	1-3-4-5-6-7-9-10-11	41
1-2-4-5-6-7-9-10-11	36	1-3-4-5-6-8-9-10-11	42
1-2-4-5-6-8-9-10-11	37	1-3-8-9-10-11	33
Critical Path: BCFHIJ. Duration 42 weeks.			



Paths	1-2-3-5-6-7	1-2-3-4-5-6-7	1-4-5-6-7
Duration(weeks)	33	34 **Critical Path: CFHIJ**	25

For completing the project as per original schedule, the project activities on the critical path should be reduced by 2 weeks. For example, we may reduce any one of the activities CFHIJ by 2 weeks or any two activities by one week each.

Illustration 9:

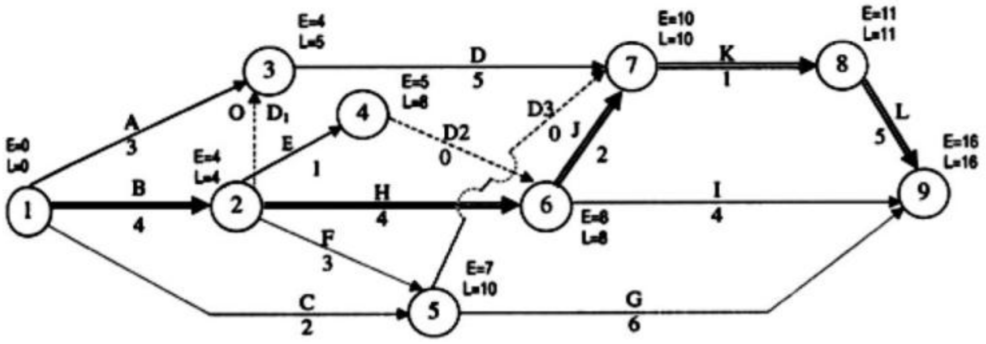
Given is the following information regarding a project:

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Dependence	-	-	-	AB	B	B	FC	B	EH	EH	CDFJ	K
Duration	3	4	2	5	1	3	6	4	4	2	1	5

Draw the Network Diagram and identify the Critical Path and Project Duration.

Answer:

Network Diagram:



Network Table:

Activity	**Duration**	**EST**	**LST**	**EFT**	**LFT**	**Total Float**	**Free Float**	**Independent Float**
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A	3	0	2	3	5	2	2 – 1= 1	1 – 0 = 1
B	4	0	0	4	4	**0**	0	0
C	2	0	8	2	10	8	8 – 3 = 5	5 – 0= 5
D1	0	4	5	4	5	1	1 – 1 = 0	0
D	5	4	5	9	10	1	1 – 0 = 1	1 – 1 = 0
E	1	4	7	5	8	3	3 – 3 = 0	0
F	3	4	7	7	10	3	3 – 3 = 0	0
G	6	7	10	13	16	3	3 – 0 = 3	3 – 3 = 0
D2	0	5	8	5	8	3	3 – 0 = 3	3 – 3 = 0
H	4	4	4	8	8	**0**	0	0
I	4	8	12	12	16	4	4 – 0= 4	4 – 0= 4
J	2	8	8	10	10	**0**	0	0
D3	0	7	10	7	10	3	3 – 0 = 3	3 – 3 = 0

K	1	10	10	11	11	**0**	0	0
L	5	11	16	16	16	**0**	0	0

Critical path is B – H – J – K – L. Expected Duration = 16 days

The columns are updated in the following order as under:

1. Activity (including Dummies) are listed from the Question and network Diagram
2. Duration (including Dummies) are listed from the Question and Network Diagram
3. EST = E value of LHS/ Tail Event from Diagram.
6. LFT = L value of RHS/ Head Event from Diagram.
5. EFT = EST + Duration as per Column (2). Hence Column (5) = Column (3) + Column (2)
4. LST = LFT - Duration as per Column (2). Hence column (4) = Column (6) – Column (2)
7. Total Float = [LET – EFT] or [LST – EST] = [Col.(6) – Col.(5)] or [Col.(4) – Col.(3)]
8. Free Float = Total Float – Head Event Slack i.e. [Col.(7) – difference between L and E of RHS Event].
   Note: If Total Float is Zero, Free Float is also equal to Zero If a negative value is derived, it is restricted to zero.
9. Independent Float = Free Float – Tail Event Slack i.e. [Col (8) – Difference between L and E of LHS Event].
   Note: If Free Float is Zero, Independent Float is also equal to Zero. If a negative value is derived, it is restricted to zero.
   Note:
   - The activities whose Total Float is Zero are Critical Activities. These Total Floats are circled and the respective activities are indicated by double in the network diagram.
   - Dummy Activities may or may not lie on the critical path. However, in this question, the dummy activities do not fall on the Critical Path.

Illustration 10:

A project with normal duration and cost along with crash duration and cost for each activity is given below:

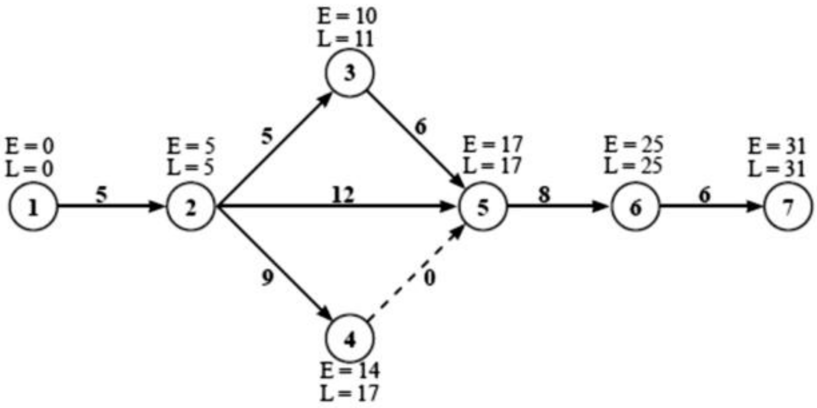
Activity	Normal time (Hrs.)	Normal cost (Rs.)	Crash time (Hrs.)	Crash cost (Rs.)
1-2	5	200	4	300
2-3	5	30	5	30
2-4	9	320	7	480
2-5	12	620	10	710
3-5	6	150	5	200
4-5	0	0	0	0
5-6	8	220	6	310
6-7	6	300	5	370

Overhead cost is Rs. 50 per hour.

Required:

Draw network diagram and identify the critical path.

Answer:



Paths →	1-2-5-6-7 (Let's denote this by A)	1-2-3-5-6-7 (Let's denote this by B)	1-2-4-5-6-7 (Let's denote this by C)
Duration	31 hours	30 hours	28 hours
The critical path is A. Its duration is 31 hours			

Illustration 11:

What are the difference between CPM and PERT.

Answer:

CPM originated from construction project while PERT evolved from R & D projects. Both CPM and PERT share the same approach for constructing the project network and for determining the critical path of the network.

There is some basic differences between PERT and CPM

PERT	CPM
1. Time estimate is probabilistic with uncertainty in time duration. Three time estimates.	1. Time estimate is deterministic with known time durations. Single time estimate
2. Event oriented	2. Activity oriented
3. Focused on time	3. Focused on time-cost trade off
4. More suitable for new projects	4. More suited for repetitive projects

Illustration 12:

Construct a network diagram satisfying the following conditions.

A < D, C; B < E; D < G, F; E, F < H; G, H, C < I

[Hint: X<Y, Z means both Y and Z cannot start until X is complete.]

Answer:

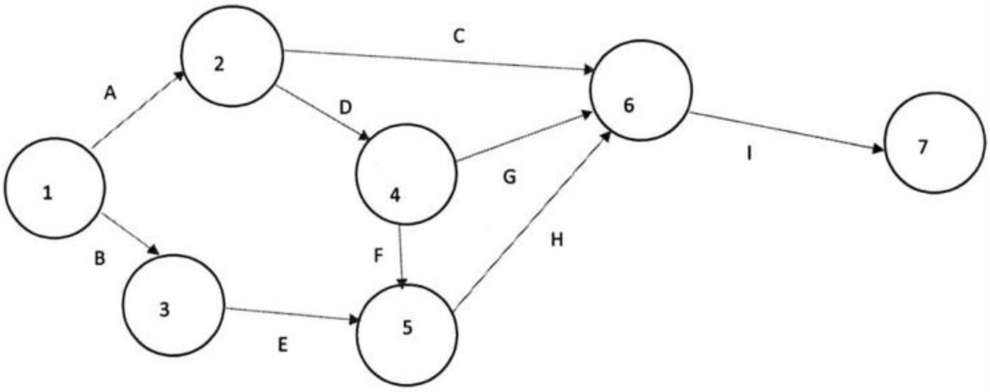


Illustration 13:

Construct the network diagram from the data given below and find

(a) total duration of the project

(b) Critical Path

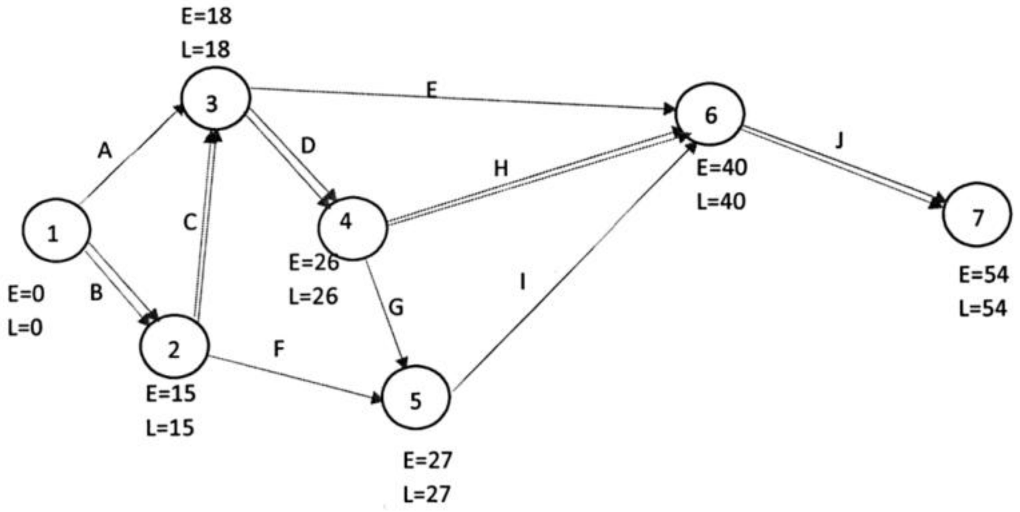
(c) EST, EFT, LST, LFT.

(d) Total float of each activity.

Activity	A	B	C	D	E	F	G	H	I	J
Duration	15	15	3	5	8	12	1	14	3	14
Predecessor Activity	-	-	B	A,C	A	B	D	D	F,G	E,H,I

Answer:

Activity (i-j)	Time (tij)	Earliest Start (ESTij)	Earliest Finish (EFTij = ESTij + tij)	Latest Start (LSTij = LFTij-tij)	Latest Finish (LFTij)	Total Float (TFij = LSTij + ESTij = LETij - EFTij)
A (1-3)	15	0	3	15	18	3
B (1-2)	15	0	0	15	15	0
C (2-3)	3	15	15	18	18	0
D (3-4)	8	18	18	26	26	0
E (3-6)	12	18	28	30	40	10
F (2-5)	5	15	32	20	37	17
G (4-5)	1	26	36	27	37	10
H (4-6)	14	26	26	40	40	0
1 (5-6)	3	27	37	30	40	10
J (6-7)	14	40	40	54	54	0



Critical Path 1-2-3-4-6-7

Critical Activity: B -C -D -H-J.

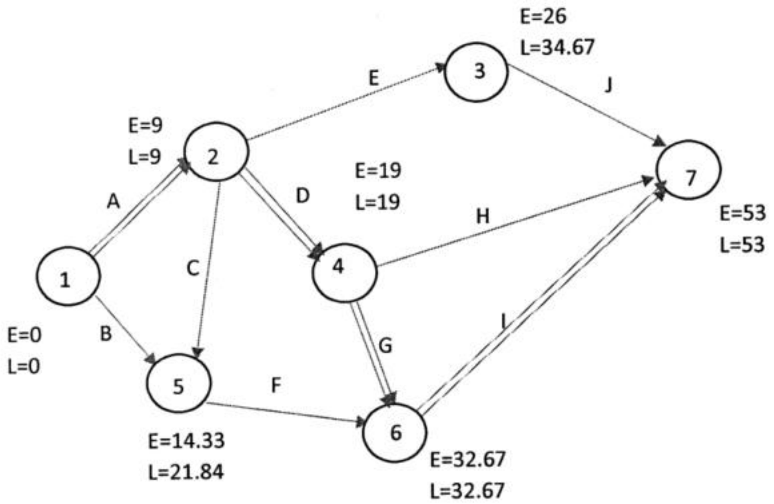
Illustration 14:

For the given data find the expected duration of the project and variance of the project.

Activity	Optimistic time (to)	Most likely Time (tm)	Pessimistic time (tp)
1-2	6	9	12
1-5	4	7	8
2-3	14	17	20
2-4	7	10	13
2-5	3	5	9
3-7	13	18	25

4-6	10	14	16
4-7	12	15	18
5-6	9	11	12
6-7	17	20	25

Answer:



Activity	Optimistic time (to)	Most likely Time (tm)	Pessimistic time (tp)	a^2= $(tp-to/6)^2$	te =to + 4tm + tp/6
1-2	6	9	12	1.00	9.0
1-5	4	7	8	0.44	6.7
2-3	14	17	20	1	17.0
2-4	7	10	13	1	10.0
2-5	3	5	9	1	5.33
3-7	13	18	25	4	18.33
4-6	10	14	16	1	13.67
4-7	12	15	18	1	15.00
5-6	9	11	12	0.25	10.83
6-7	17	20	25	1.78	20.33

The critical path is 1 - 2 - 4 - 6 - 7

Variance of the critical path = 1 + 1 + 1.78 = 3.78

SD of the critical path = SD of the network diagram = $\sqrt{(3.78)}$ = 1.944

Illustration 15:

A marketing organization is planning a questionnaire survey on behalf of their client to assess market potential of instant foods. The following activities are involved in this project:

Task	Duration(days)			
	Precedence	Optimistic	Most(likely)	Pessimistic
A. Design Questionnaire		2	3	4
B. Sample design		6	10	20
C. Testing of Questionnaire and refinements		2	4	6
D. Recruiting interviewers	B	2	3	10
E. Training of Interviewers	D, A	1	1	1

F. Allocation of Interviewers to territories	B	4	5	6
G. Conducting Interviews	C, E, F	5	12	25
H. Evaluation of results	G	6	10	20

(a) Find the expected duration and variance of each task.

(b) Draw an arrow diagram (network) of the project.

(c) Calculate EST, EFT, LST, LFT & TF

(d) Identify the critical path.

(e) Find the critical path duration of the project.

(f) What percentage of the project will be complete in 44 days?

Activity	A	M	B	Te	Variance	EST	EFT	LST	LFT	TF
A	2	3	4	3	1/9	0	3	12	15	12

(g) Find the no of day by which approximately 100% of the project will be completed

Answer:

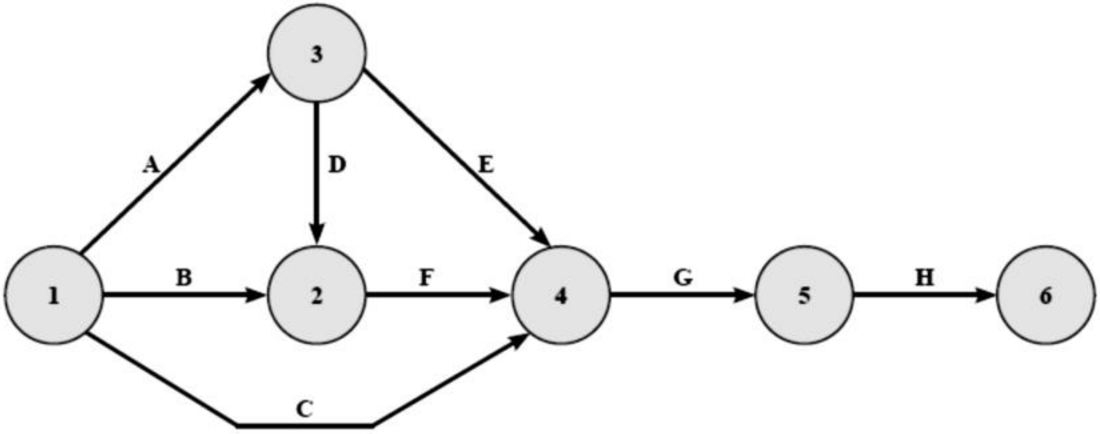
t_e = Expected time

A = Optimistic time;

M = Most likely time;

B = Pessimistic time

Activity	**A**	**M**	**B**	**Te**	**Variance**	
A	2	3	4	3	1/9	**t_e=(A + 4M + B) / 6** **Variance (t) = $[(B-A)]^2$**
B	6	10	20	11	49/9	
C	2	4	6	4	4/9	
D	2	3	10	4	16/9	
E	1	1	1	1	0	
F	4	5	6	5	1/9	
G	5	12	25	13	100/9	
H	6	10	20	11	49/9	



Activity	**A**	**M**	**B**	**Te**	**Variance**	**EST**	**EFT**	**LST**	**LFT**	**TF**
A	2	3	4	3	1/9	0	3	12	15	12
B	6	10	20	11	49/9	0	11	0	11	0
C	2	4	6	4	4/9	0	4	12	16	12
D	2	3	10	4	16/9	11	15	11	15	0
E	1	1	1	1	0	15	16	15	16	0
F	4	5	6	5	1/9	11	16	11	16	0
G	5	12	25	13	100/9	16	29	16	29	0

H	6	10	20	11	49/9	29	40	29	40	0

There are two Critical Path

(i) 1---2---3---4---5---6 = (11 + 4 + 1 + 13 + 11) = 40

(ii) 1---2---4---5---6 = (11 + 5 + 13 + 11) = 40

As both the critical path suggest for both cases 40 days required to complete the project, so we calculate the

standard deviation of critical path

$$\mathbf{CSD_1} = \sqrt{\frac{49}{9} + \frac{16}{9} + 0 + \frac{100}{9} + \frac{49}{9}} = \frac{14.9}{3} = 4.966 \text{ (Approx.)}$$

$$\mathbf{CSD_2} = \sqrt{\frac{49}{9} + \frac{1}{9} + \frac{100}{9} + \frac{49}{9}} = \frac{14.1}{3} = 4.7 \text{ (Approx.)}$$

Here, CSD_2 performing better than CSD1, so we select the 2nd Critical Path.

Then here, μ = 40 , σ = 4.7

Since in 44 day taken then the percentage of work done

P(T = 44) = P((T-μ)/σ< = ((44-40)/4.7))

P(Z< = 4/4.7)

P(Z< = 0.8) = 0.78814

Nearly 79% of the project will be completed during 44 days.

For the completion of 100% of the project we can take the 3 sigma limit

P(T<=n)

P((T-μ)/σ< = (n-μ)/σ) = P(Z<= 3)

P(Z< = (n-40)/4.7) = P(Z<= 3)

n = 4.7 × 3 + 40

n = 54 day (Approx)

Illustration 16:

A management institute plans to organize a conference on use of "Operation Research for decision making". In order to co-ordinate the project, it has decided to use a PERT network. The major activities and time estimates for activity has been compiled as follows:

Sl.No.	Activity description	Time estimate (a-m-b)	Activity that must precede
A	Design conference meeting theme	1-2-3	None
B	Design front cover of conference proceedings	1-2-3	A
C	Design brochure	1-2-3	A
D	Compile list of distinguished speakers	2-4-6	A
E	Finalize brochure and print it	2-5-14	C and D
F	Make travel arrangements for distinguished speakers	. 1-2-3	D
G	Send brochures	1-3-5	E
H	Receive papers for conference	10-12-20	G
I	Edit papers	3-5-7	H
J	Print proceedings	5-10-15	B and I

(a) Draw the network.

(b) Calculate expected time for each activity and variance for each activity.

(c) Calculates EST, EFT, LST, LFT, TF

(d) Identify critical path.

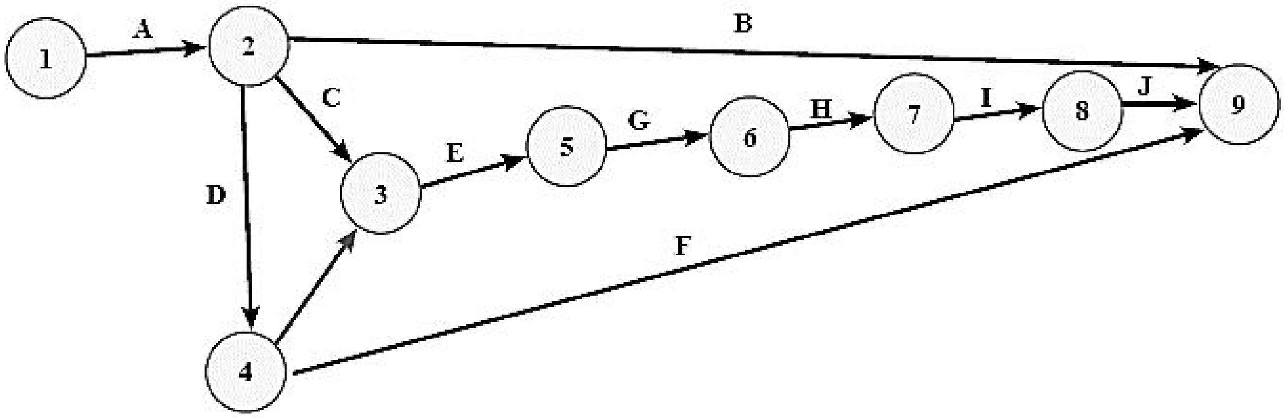
(e) Find the no of day by which approximately 90% of the project will be completed

Answer:

t_e = Expected time
A = Optimistic time;
M = Most likely time;
B = Pessimistic time

Activity	A	M	B	te	Variance (t)
A	1	2	3	2	1/9
B	1	2	3	2	1/9
C	1	2	3	2	1/9
D	2	4	6	4	4/9
E	2	5	14	6	4
F	1	2	3	2	1/9
G	1	3	5	3	4/9
H	10	12	20	13	25/9
I	3	5	7	5	4/9
J	5	10	15	10	25/9

t_e=(A + 4M + B)/6
Variance (t) = $[(B - A) / 6]^2$



Activity	A	M	B	te	Variance (t)	EST	EFT	LST	LFT	TF
A	1	2	3	2	1/9	0	2	0	2	0
B	1	2	3	2	1/9	2	4	41	43	39
C	1	2	3	2	1/9	2	4	4	6	2
D	2	4	6	4	4/9	2	6	2	6	0
E	2	5	14	6	4	6	12	6	12	0
F	1	2	3	2	1/9	6	8	41	43	35
G	1	3	5	3	4/9	12	15	12	15	0
H	10	12	20	13	25/9	15	28	15	28	0
I	3	5	7	5	4/9	28	33	28	33	0
J	5	10	15	10	25/9	33	43	10	43	0
Dummy						6	6	6	6	6

Critical Path = 1 - 2 - 4 - 3 - 5 - 6 - 7 - 8 - 9
= (2 + 4 + 6 + 3 + 13 + 5 + 10) = 43

$$CSD= \sqrt{\frac{1}{9}+\frac{4}{9}+4+\frac{4}{9}+\frac{25}{9}+\frac{4}{9}+\frac{25}{9}} = 3.31$$

Let n be the no. of days by which 90% of the project will be completed.
P[T<= n] = 0.90

$P[(T-43)/3.31<=(n-43)/3.31\} = 0.90 = 1.28$

$(n-43)/3.31 = 1.28$

$n = (1.28 \times 3.31) + 43$

$n = 47.23 = 48$ days

7. Economics of maintenance and Spare parts management

Questions For Classroom Discussion

Break Down Maintenance or Preventive Maintenance

Question 1

The following data relating to the number of breakdown in a firm for the last 300 days

Number of Breakdown	Frequency
0	40
1	150
2	70
3	30
4	10
Total	300

The firm estimates that each breakdown costs Rs 650 and is considering adopting a preventive maintenance program, which would cost Rs 200 per day and limit the number of break down to an average of one per day.

What is the expected annual saving from preventive maintenance program?

Question 2

The number of breakdowns of an equipment over the past 2 years is given below.

Number of Breakdown	No of Month this Occurred
0	3
1	7
2	9
3	3
4	2
Total	24

Each breakdown cost an average of 300. Preventive maintenance service can be hired at a cost of Rs 150 per month and it will limit the number of breakdowns to an average of one per month.

Which maintenance is preferred?

Preventive or Breakdown?

Obsolescence and Replacement

$$T(n) = C-S + \sum m$$

$$A(n) = \frac{[C-S+ \sum m]}{n}$$

Question 3

A firm is using a machine whose purchase price is Rs.13,000. The installation charges amount to Rs.3,600 and the machine has a scrap value of only Rs.1,600 because the firm has a monopoly of this type of work. The maintenance cost in various years is given in the following table.

Year	Cost
1	250
2	750
3	1000
4	1500
5	2100
6	2900

7	4000
8	4800
9	6000

The firm wants to determine after how many years should the machine be replaced on economic considerations, assuming that the machine replacement can be done only at the year ends.

Question 4

The Simple Engineering Company has a machine whose purchase price is Rs 80,000. The expected maintenance costs and resale price in different years are as given here:

Year	Maintenance Cost	Re Sale Price
1	1000	75000
2	1200	72000
3	1600	70000
4	2400	65000
5	3000	58000
6	3900	50000
7	5000	45000

After what time interval, in your opinion , should the machine be replaced?

Question 5

A machine M1, costing Rs.9,000 has a maintenance cost of Rs.200 in the first year of its operation which rises by Rs.2,000 in each of the successive years.

Assuming that the machine replacement can be done only at the end of a year, determine the best age at which the machine be replaced.

Group Replacement

Question 6

A company has 50 identical machines in its facilities. The cost of preventive servicing (CP) is Rs 20, and the cost of repair after breakdown (CR) is Rs 100. The company seeks the minimum cost preventive servicing frequency and has collected the data on breakdown probabilities in the following table:

Months after Servicing that Breakdown	Probability that Breakdown will occur	Expected Number
1	0.10	
2	0.05	
3	0.05	
4	0.10	
5	0.15	
6	0.15	
7	0.20	
8	0.20	
		5.40

Question 7 **Home Work – Previous Year Question (2+2+5 =9 Marks)**

The Management of Green in , Hotel is considering the periodic replacement of light bulbs fitted in its rooms. There are 500 rooms in the hotel and each room has 6 bulbs. The management of Green in hotel is now following the policy of replacing bulbs as they fails at a cost of RS 30 per bulb. The management feels that this cost can be reduced to Rs 10 by adopting periodic replacement method ?

The following mortality rate has been observed for the said light bulbs.

Assume that the bulbs that fails during the month are replaced just before the end of that month.

Months of Use	1	2	3	4	5
Probability bulbs failing by the end of the month	0.10	0.15	0.25	0.30	0.20

Required:

1) Compute the costs of (in Rs) of Individual Replacement of Bulbs per month
2) Present the optimum Replacement cycle under Group Replacement
3) State the best strategy for the management of Green Hills to follow

(Present Calculation to the nearest integer)

Independent Problems

Question 8

Indian Electronics, manufactures TV sets, carries out the picture tube testing for 2000 hours.

A sample of 100 tubes was put through this quality test during which two tubes failed.

1) If the average usage of TV by the customer is 4 hours/day and if 10,000 TV sets were sold, then in one year how many tubes were expected to fail and
2) What is the mean time between failures for these tubes?

Question 1

The main shaft of an equipment has a very high reliability of 0.990.

The equipment comes from Russia and has a high downtime cost associated with the failure of this shaft. This is estimated at Rs.20,000,000 as the costs of sales lost and other relevant costs. However, this spare is quoted at Rs.1,000,000 at present.

Should the shaft spare be procured along with the equipment and kept or not?

Question 2

Product A has a Mean Time Between Failures (MTBF) of 30 hours and has a Mean Time To Repairs (MTTR) of 5 hours. Product B has a MTBF of 40 hours and has a MTTR of 2 hours.

Q.1 Which product has the higher reliability?

Q.2 Which product has greater maintainability?

Q.3 Which product has greater availability ?

Illustration From Study Material

Illustration 1:

M/s Nirmala Toolkit Pvt. Ltd. has a workshop comprising of 20 tool machines of similar type. To improve the preventive maintenance plan, the workshop manager collects the data of failure history of the machines as under

Elapsed time after Maintenance attention (in month)	Probability of failure
1	0.20
2	0.15
3	0.15
4	0.15
5	0.15
6	0.20

It costs Rs. 150 to attend a failed machine and rectify the same. Compute the yearly cost of servicing the broken down machines.

Answer:

Expected time before failure.

= 0.20 × 1 + 0.15 × 2 + 0.15 × 3 + 0.15 × 4 + 0.15 × 5 + 0.20 × 6 = 3.5 months

Therefore number or repair/machine/annum = 12/3.5

Considering 20 machines and Rs. 150 to attend a failed machine the yearly cost of servicing

= 12/3.5 × 20 × 150 = Rs. 10286.

Illustration 2:

A Public transport system is experiencing the following number of breakdowns for months over the past 2 years in their new fleet of vehicles:

Number of breakdowns	0	1	2	3	4
Number of months this occurred	2	8	10	3	1

Each break down costs the firm an average of Rs. 2,800. For a cost of Rs. 1,500 per month, preventive maintenance can be carried out to limit the breakdowns to an average of one per month. Which policy is suitable for the firm?

Answer:

Converting the frequencies to a probability distribution and determining the expected cost/month of breakdowns we get:

No. of breakdowns (x)	**Frequency in months (f)**	**Probability (p = f/Σf)**	**Expected no. of breakdowns (px)**
0	2	0.083	0.000
1	8	0.333	0.333
2	10	0.417	0.834
3	3	0.125	0.375
4	1	0.042	0.168
	Σf = 24	Σp = 1	Total 1.710 = Σpx

Expected Breakdown cost per month; Expected no. of breakdowns per month × cost of each breakdown = 1.710 × Rs. 2800 = Rs. 4788.

Preventive maintenance cost per month: -

Average cost of one breakdown/month	= Rs. 2, 800
Maintenance contract cost/month	= Rs. 1,500
Total	= Rs. 4,300

Thus, preventive maintenance policy is suitable for the firm.

Illustration 3:

Indian Electronics, manufactures TV sets and carries out the picture tube testing for 2000 hours. A sample of 100 tubes was put through this quality test during which two tubes failed. If the average usage of TV by the customer is 4 hours/day and if 10,000 TV sets were sold, then in one year how many tubes were expected to fail and what is the mean time between failures for these tubes?

Answer:

The total test time = (100 tubes) × 2000 hours = 200,000 tube-hours.

There are two tubes which have failed and hence the total time is to be adjusted for the number of hours lost due to the failures during the testing.

The lost hours are computed as = 2 × 2000 / 2 = 2000 hours.

The assumption is made here is that each of the failed tubes have lasted an average of half of the test period. Therefore, the test shows that there are two failures during (2,00,000 – 2000) = 1,98,000 tube hours of testing. During 365 days a year (four hours a day) for 10,000 tubes the number of expected failures

$$\frac{1,98,000}{2} \times 10,000 \times 365 \times 4 = 147.47 = 148 \text{ tubes approximately.}$$

$$\text{Mean time between failures} = \frac{\text{1,98,000 tubes hrs. of testing}}{\text{2 failure}}$$

$$= \text{99,000 tubes hours per failure} = \frac{99,000}{4 \times 365} = \text{67.8 tubes year per failure}$$

Illustration 4:

M/s XYZ Pvt. Ltd has 50 identical machines in its facilities. The company has the recorded figure for cost of preventive maintenance (Cp) and cost of breakdown maintenance (Cb) as Rs. 20 and Rs. 100 respectively. The company wants to reduce the breakdown occurrence while minimizing Cp. Given is the data on breakdown occurance.

Probabilities of machine breakdown, by month:

Months after servicing that breakdown occurs (i)	Probability that breakdown will occur (Pi)	$i.P_i$
1	0.10	0.10
2	0.05	0.10
3	0.05	0.15
4	0.10	0.40
5	0.15	0.75
6	0.15	0.90
7	0.20	1.40
8	0.20	1.60
Total	1.00	5.40

Answer:

The mean time before failure is 5.4 months and the expected cost with no preventive maintenance would be 100 × 50 / 5.4 = Rs. 925.93 per month. The following calculations show B_j, the expected number of breakdowns between preventive maintenance intervals, for the possible intervals, that may be considered.

B_1 = MP1 = 50 (0.10) = 5

B_2 = m (P1 + P2) + B1P1 = 50(0.10+0.05) + 5(0.10) = 8

B_3 = 50 (0.10 + 0.05 + 0.05) + 8 (0.10) + 5 (0.05) = 11.05

Accordingly, B_4 = 16.75, B_5 = 25.63, B_6 = 35.5, B_7 = 48.72, B_8 = 63.46.

The costs of various preventive maintenance intervals are summarised in the table below :

Cost of alternative preventive maintenance intervals –

Number of months between preventive services (j)	**Bj Expected Number of Breakdown in j months**	**Expected cost/ month to Repair Breakdown CR×Bj/j**	**Cost per month for preventive service every j month CR(M)/j**	**Total expected cost per month of preventive maintenance and repair**
(1)	**(2)**	**(3)**	**(4)**	**(5)**
1	5.00	500.00	1000.00	1500.00
2	8.00	400.00	500.00	900.00
3	11.05	368.33	333.33	701.66
4	16.75	418.75	250.00	668.75
5	25.63	512.60	200.00	712.60
6	35.50	591.67	166.67	758.34
7	48.72	696.00	142.86	838.86
8	63.46	793.25	125.00	918.25

A policy of performing preventive maintenance every 4 months results in the lowest average cost, about Rs. 669.

This amount is Rs. 257 per month less than the Rs. 926 expected cost without preventive maintenance. This policy would reduce the costs by (257 ÷ 926) × 100 = 27.75% below the cost of repairing the machines only when they breakdown.

Illustration 5:

Assume the following three breakdown probability distribution

Month following Maintenance	Probability of Breakdown		
	(1)	(2)	(3)
1	0.5	0.1	0.1
2	0.1	0.1	0.1
3	0.1	0.1	0.5
4	0.1	0.1	0.1
5	0.1	0.2	0.1
6	0.1	0.4	0.1

Which, if any, of these distributions lend themselves to a preventive maintenance program? Why?

Answer:

Policy 1:

Month following Maintenance (i)	**Probability of Breakdown (p)**	**Average free run time (i * p)**
1	0.5	0.5
2	0.1	0.2
3	0.1	0.3
4	0.1	0.4
5	0.1	0.5
6	0.1	0.6
		Σ2.5months/breakdown/machine

Therefore the average number of breakdowns for the pool of say 100 machines per month will be:

For 1 machine in 2.5 months 1 breakdown

So for 1 machine in 1 month (1/2.5) breakdown

So for 100 machines in 1 month (100/2.5) = 40 breakdowns

Policy 2:

Month following Maintenance (i)	**Probability of Breakdown (p)**	**Average free run time (i * p)**
1	0.1	0.1
2	0.1	0.2
3	0.1	0.3
4	0.1	0.4
5	0.2	1.0
6	0.4	2.4
		Σ4.4months/breakdown/machine

Therefore the average number of breakdowns for the pool of say 100 machines per month will be:
For 1 machine in 4.4 months 1 breakdown
So for 1 machine in 1 month (1/4.4) breakdown
So for 100 machines in 1 month (100/4.4) = 22.73 breakdowns

Policy 3:

Month following Maintenance (i)	**Probability of Breakdown (p)**	**Average free run time (i * p)**
1	0.1	0.1
2	0.1	0.2
3	0.5	1.5
4	0.1	0.4
5	0.1	0.5
6	0.1	0.6
		Σ3.3months/breakdown/machine

Therefore the average number of breakdowns for the pool of say 100 machines per month will be:
For 1 machine in 3.3 months 1 breakdown
So for 1 machine in 1 month (1/3.3) breakdown
So for 100 machines in 1 month (100/3.3) = 30.30 breakdowns

Preventive maintenance programs are generally applicable to breakdown distributions with low variability. Policy 2 has the lowest variability as no of breakdowns in a month for a pool of say 100 machines are 22.73---the lowest among three policies.

Therefore we may conclude that policy 2 could lead to a preventive maintenance program.

Illustration 6:

Assume the following three breakdown probability distribution

Month following Maintenance	Probability of Breakdown
1	0
2	0.1
3	0.1
4	0.1
5	0.2
6	0.5

Let us take Average Repair Cost on breakdown C_R = Rs.100 & Cost of Preventive maintenance C_{PM} = Rs.75, Cost of Individual Replacement CI = Rs.80, Cost of Group Replacement = Rs.50 / machine
For a pool of 100 machines, Could you recommend PM? When you will go for Replacement?

Answer

Month following Maintenance (i)	**Probability of Breakdown (p)**	**Average free run time (i * p)**
1	0.0	0.0
2	0.1	0.2
3	0.1	0.3
4	0.1	0.4
5	0.2	1.0
6	0.5	3.0
		Σ4.9 months/breakdown/machine

Therefore the average number of breakdowns for the pool of say 100 machines per month will be:

For 1 machine in 4.9 months 1 breakdown

So for 1 machine in 1 month (1/4.9) breakdown

So for 100 machines in 1 month (100/4.9) = 20.40816 breakdowns

Repair Policy Cost = Average number of repairs per month × Average repair cost on breakdown

= 20.40816 × 100 = Rs.2,040.816

Preventive Maintenance Costs for the Six Preventive Maintenance Cycles:

Table-I

Preventive Maintenance Cycle (n) , months	**Expected Breakdowns in PM Cycle**	**Average No of Breakdowns per month (Col.2/ Col.1)**	**Expected Monthly Breakdown Cost (Col.3 x Rs.100)**	**Expected Monthly PM Cost (Rs.75 x 100)/ Col.1**	**Expected Monthly Cost of each PM cycle (Col.4 + Col.5)**
1	0	0	0.00	7500	7500.00
2	10	5	500.00	3750	4250.00
3	20	6.667	666.70	2500	3166.70
4	31	7.75	775.00	1875	2650.00
5	53	10.6	1060.00	1500	2560.00
6	106.1	17.683	1768.30	1250	3018.30

Computation of Col. 2:

Month 1: 100*0.0 = 0

Month 2: 100*(0.0+0.1) +0*0.0 = 10

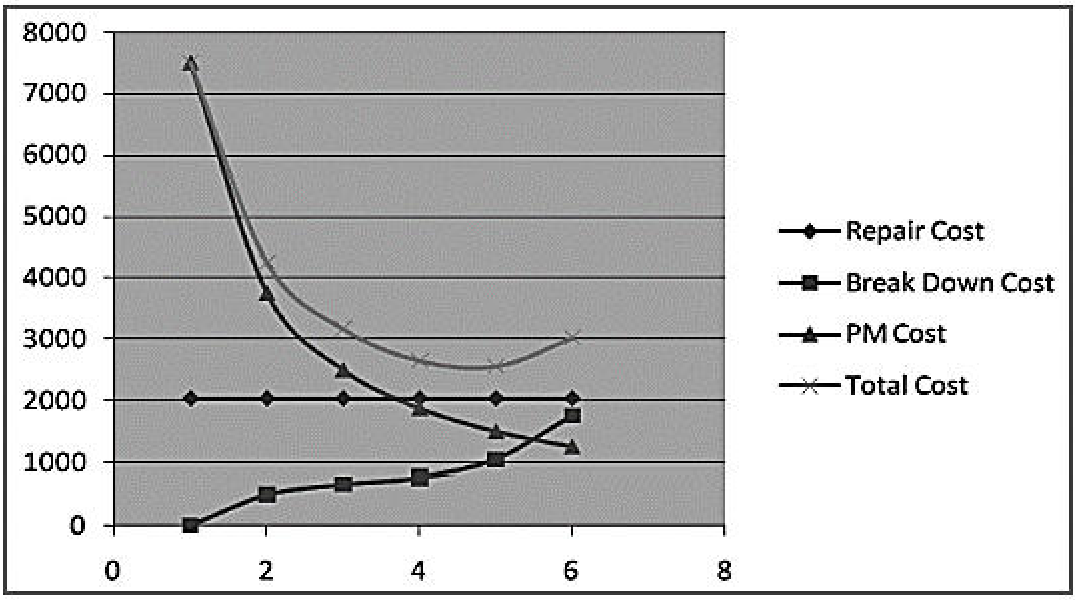
Month 3: 100*(0.0+0.1+0.1) + 0*0.1 +10*0.0 = 20

Month 4: 100*(0.0+0.1+0.1+0.1) + 0*0.1+10*0.1+20*0.0 = 31

Month 5: 100*(0.0+0.1+0.1+0.1+0.2) +0*0.1+10*0.1+20*0.1+31*0.0 = 53

Month 6: 100*(0.0+0.1+0.1+0.1+0.2+0.5) +0*0.2+10*0.1+20*0.1+31*0.1+53*0.0 = 106.1

Graphical Representation Policy 1:



So from the above it is clearly observed that PM policy is inferior to Repair policy. But will repair policy sustainable?

Answer is NO. After continuing for some time with repair policy cost effectiveness of the policy will be lost and at this stage we have to replace ---either individual machines or in blocks.

To do this analysis we will follow the steps below mentioned:

Step-I: Determination of Number of failures in different weeks.

Table-II

Preventive Maintenance Cycle (n), months	**Probability of Breakdown (*p*)**	**Expected Breakdowns in PM Cycle**
1	0.0	0
2	0.1	10
3	0.1	20
4	0.1	31
5	0.2	53
6	0.5	106.1

Column 2 of Table 1

Step-2: Determination of Average Cost of Different Policies

Table-III

Months	**No of Individual Replacements**	**Cost of Replacements**			
		Individual (Col2 x 80)	**Group (100 X 50)**	**Total (Col3 +Col4)**	**Average Cost (Col5/Col1)**
1	0	0	5000	5000	5000
2	10	800	5000	5800	2900
3	20	1600	5000	6600	2200
4	31	2480	5000	7480	1870
5	53	4240	5000	9240	1848
6	106.1	8488	5000	13488	2248

From the table it is observed that the minimum cost per month is obtained by replacing all the machines (whether failed or not) after every 5 months. Thus optimal replacement time interval = 5 months.

But we can go for a policy “Replace as and when a machine fail” and in that case there will not be any group replacement.

To check the feasibility of “Replace as and when a machine fails” the computation will be as following:

Life (months)	Mean value (Xi)	Probability (pi)	pi x Xi
0-1	0.5	0.0	0
1-2	1.5	0.1	0.15
2-3	2.5	0.1	0.25
3-4	3.5	0.1	0.35
4-5	4.5	0.2	0.9
5-6	5.5	0.5	2.75
			4.4

Mean life of a machine is = 4.4

Expected no of failures of a machine during a week = No of Machines/ Mean life of a machine
= 100/4.4 = 22.727

Weekly replacement cost = Expected no of replacements X cost of replacements
= 22.727 X 80
= 1818.16

Since the cost of the policy of individual replacement i.e. "Replace as and when a machine fail" is less than that of the group replacement, it is advisable to go for individual replacement.

Illustration 7:

Refer Illustraton No. 5. Let us take Average Repair Cost on breakdown CR = Rs.90 & Cost of Preventive maintenance CPM = Rs.30

Could you prove your conclusion given in A1 for a pool of 100 machines?

Answer:

Repair Policy Cost of Policy 1 = Average number of repairs per month × Average repair cost on breakdown = 40 × 90 = Rs.3,600.

Data taken from Solution 5.

Preventive Maintenance Costs for the Six Preventive Maintenance Cycles:

Table-I

Preventive Maintenance Cycle (n), months	**Expected Breakdowns in PM Cycle**	**Average No of Breakdowns per month (Col.2/ Col.1)**	**Expected Monthly Breakdown Cost (Col.3 × Rs.90)**	**Expected Monthly PM Cost (Rs.30 × 100)/ Col.1**	**Expected Monthly Cost of each PM cycle (Col.4 + Col.5)**
1	50.00	50.00	4500.00	3000	7500.00
2	85.00	42.50	3825.00	1500	5325.00
3	117.50	39.17	3525.30	1000	4525.30
4	152.25	38.06	3425.40	750	4175.40
5	191.38	38.28	3445.20	600	4045.20
6	236.16	39.36	3542.40	500	4042.40

Computation of Col. 2:

Month 1: 100*0.5 = 50

Month 2: 100*(0.5+0.1) +50*0.5 = 85

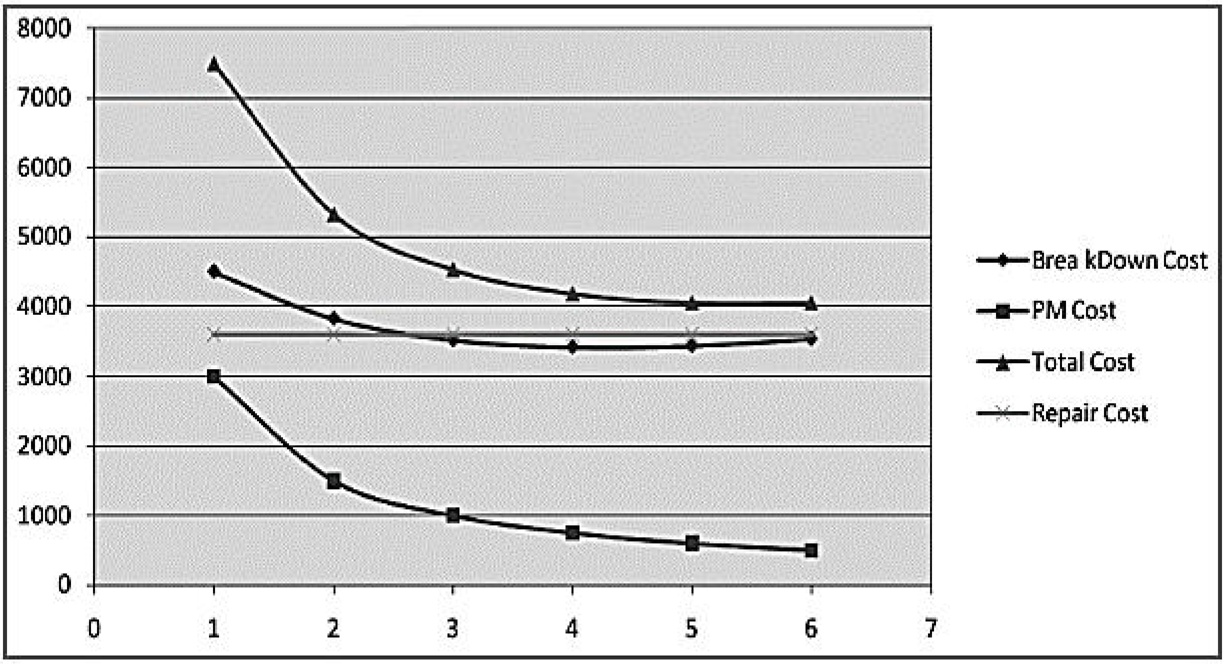
Month 3: 100*(0.5+0.1+0.1) + 50*0.1 +85*0.5 = 117.5

Month 4: 100*(0.5+0.1+0.1+0.1) + 50*0.1+85*0.1+117.5*0.5 = 152.25

Month 5: 100*(0.5+0.1+0.1+0.1+0.1) +50*0.1+85*0.1+117.5*0.1+152.25*0.5 = 191.38

Month 6: 100*(0.5+0.1+0.1+0.1+0.1+0.1) +50*0.1+85*0.1+117.5*0.1+152.25*0.1+191.38*0.5 = 236.16

Graphical Representation Policy 1:



Repair Policy Cost of Policy 2 = Average number of repairs per month × Average repair cost on breakdown = 22.73 × 90 = Rs.2,045.7 (Data taken from Solution 5).

Preventive Maintenance Costs for the Six Preventive Maintenance Cycles

Preventive Maintenance Cycle (n), months	Expected Breakdowns in PM Cycle	Average No of Breakdowns per month (Col.2/Col.1)	Expected Monthly Breakdown Cost (Col.3 × Rs.90)	Expected Monthly PM Cost (Rs.30 × 100)/ Col.1	Expected Monthly Cost of each PM cycle (Col.4 + Col.5)
1	10.00	10.00	900.00	3000	3900.00
2	21.00	10.50	945.00	1500	2445.00
3	33.10	11.03	992.70	1000	1992.70
4	46.41	11.60	1044.00	750	1794.00
5	71.05	14.21	1278.90	600	1878.90
6	119.16	19.86	1787.40	500	2287.40

Computation of Col. 2:

Month 1: 100*0.1 = 10

Month 2: 100*(0.1+0.1) +10*0.1 = 21

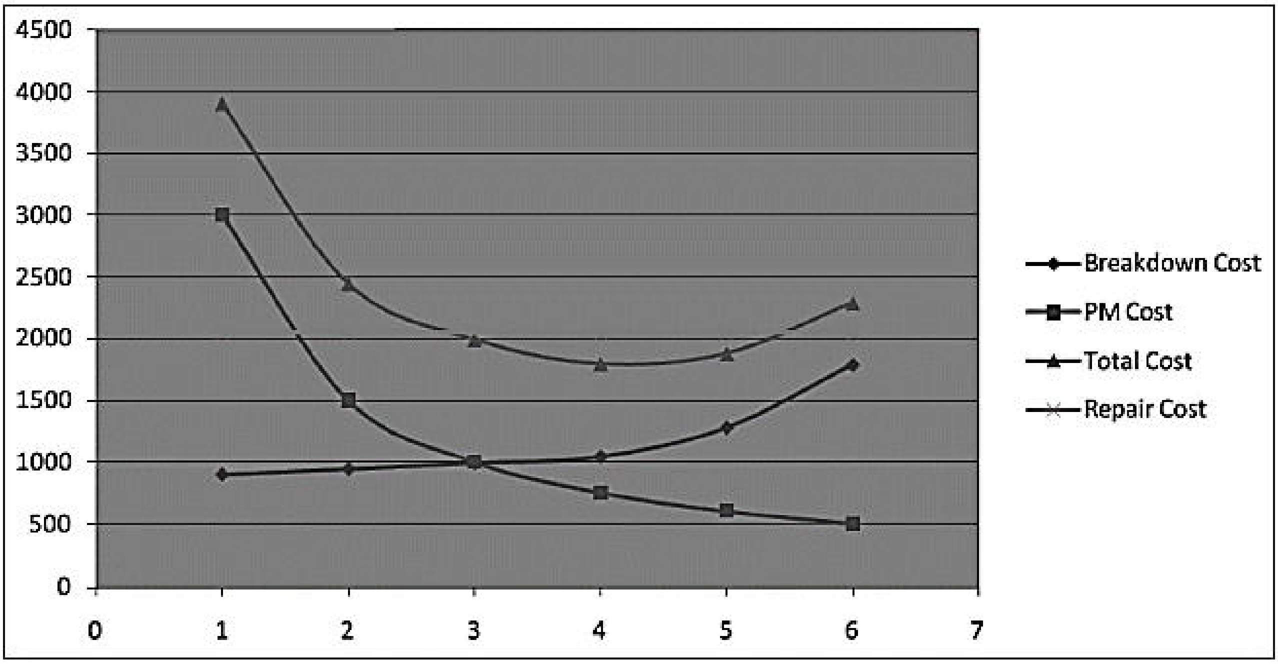
Month 3: 100*(0.1+0.1+0.1) + 10*0.1 +21*0.1 = 33.1

Month 4: 100*(0.1+0.1+0.1+0.1) + 10*0.1+21*0.1+33.1*0.1 = 46.41

Month 5: 100*(0.2+0.1+0.1+0.1+0.1) +10*0.1+21*0.1+33.1*0.1+46.41*0.1 = 71.05

Month 6: 100*(0.4+0.2+0.1+0.1+0.1+0.1)+10*0.2+21*0.1+33.1*0.1+46.41*0.1+71.05*0.1= 119.16

Graphical Representation Policy 2:



Repair Policy Cost of Policy 3 = Average number of repairs per month × Average repair cost on breakdown = 30.30 × 90 = Rs.2,727 (Data taken from Ans 1)

Preventive Maintenance Costs for the Six Preventive Maintenance Cycles

Preventive Maintenance Cycle (n), months	**Expected Breakdowns in PM Cycle**	**Average No of Breakdowns per month (Col.2/Col.1)**	**Expected Monthly Breakdown Cost (Col.3 x Rs.90)**	**Expected Monthly PM Cost (Rs.30 x 100)/ Col.1**	**Expected Monthly Cost of each PM cycle (Col.4 + Col.5)**
1	10.00	10.00	900.00	3000	3900.00
2	21.00	10.50	945.00	1500	2445.00
3	73.10	24.37	2193.30	1000	3193.30
4	94.41	23.60	2124.00	750	2874.00
5	118.25	23.65	2128.50	600	2728.50
6	160.92	26.82	2413.80	500	2913.80

Computation of Col. 2:

Month 1: 100*0.1 = 10

Month 2: 100*(0.1+0.1) +10*0.1 = 21

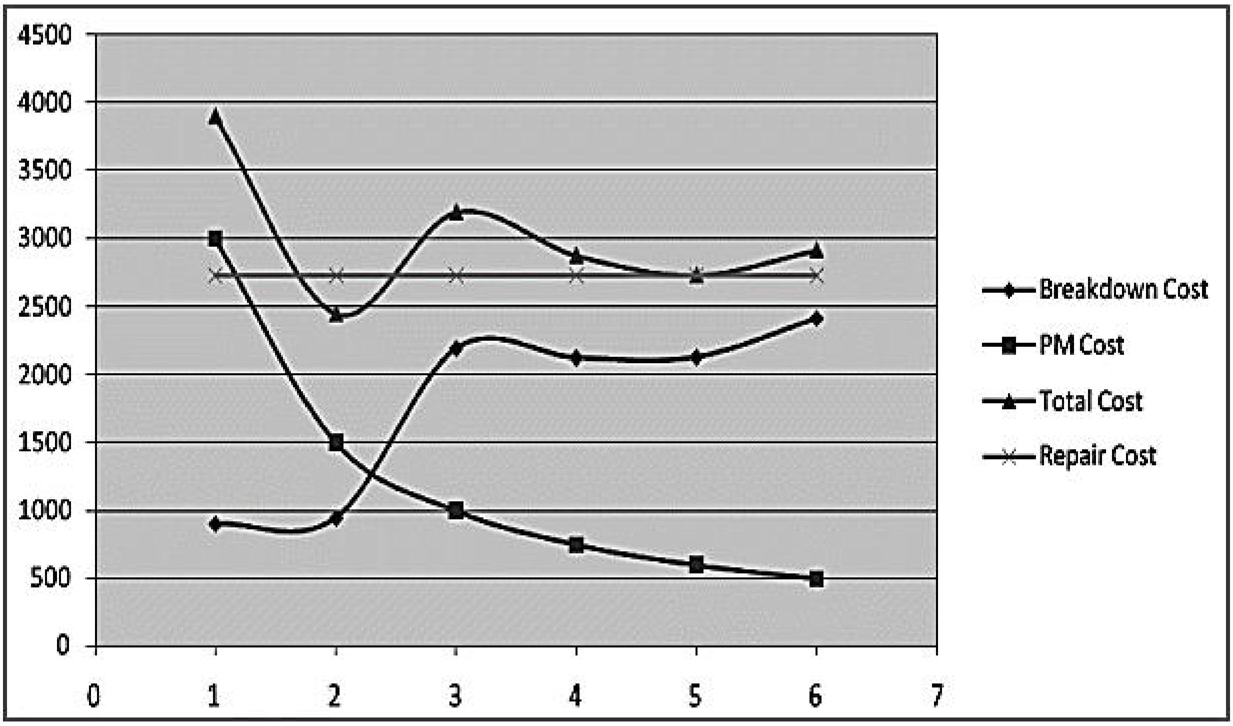
Month 3: 100*(0.5+0.1+0.1) + 10*0.1 +21*0.1 = 73.1

Month 4: 100*(0.1+0.5+0.1+0.1) + 10*0.5+21*0.1+73.1*0.1 = 94.41

Month 5: 100*(0.1+0.1+0.5+0.1+0.1) +10*0.1+21*0.5+73.1*0.1+94.41*0.1 = 118.25

Month 6: 100*(0.1+0.1+0.1+0.5+0.1+0.1)+10*0.1+21*0.1+73.1*0.5+94.41*0.1+118.25*0.1 = 160.92

Graphical Representation Policy 3:



If we refer three graphs it is clear that –

Under Policy 1 (Fig –I) Repair cost Rs.3,600 is always less than cost of all PM cycles - Refer Col.6 of Table-I. Therefore if breakdown probability distribution is like under Policy 1, management will opt for policy of repairing machine when it breaks down.

Under Policy 2 (Fig –II) PM cycle of 4 months with the cost of Rs.1,794 - Refer Col.6,Row 4 of Table-II, is less than Repair cost Rs.2,045.7. Therefore, if breakdown probability distribution is like under Policy 2, management will opt for PM policy of 4 months instead of going for policy of repairing machine when it breaks down. This way management can save Rs.251.7.

On similar logic under policy 3 PM is preferable to Repair as and when required policy. But in comparison to policy 2, policy 3 is inferior as Repair cost under policy 2- Rs.2,045.7 < Repair cost under policy 3- Rs.2,727.

PM policy Cost under policy 2- Rs.1,794 < PM policy Cost under policy 3- Rs.2,445.

The decision concerning preventive maintenance versus Repair depends on i) factor costs C_R and C_{PM} ii) the breakdown probability distribution; besides other sensitivities.

Illustration 8:

Compute the requirement of spares for breakdown maintenance for an item that exhibits a Poissonian behavior for failure rates with a mean breakdown rate of five items per month. If the lead time for procuring these spares is one month and a service level of 90 per cent is to be used, what buffer stock of these items should be maintained? (A fixed re-order quantity system of inventory is being used).

Answer:

Buffer stock is required to cover the lead time only, i.e. to cover one month's period.

Mean consumption rate = 5 per month

Referring to the Poisson distribution table for a = 5, we have for

x = 7. ... Cumulative probability = 0.867

x = 8 ... Cumulative probability = 0.932

Thus, with seven items only 86.7 per cent service level is attained; with eight items 93.2 per cent service level is obtained. Since one would err on the higher side of the service level, the value of x = 8 is chosen.

This means, the amount of spares stock that has to be kept must correspond to a maximum demand rate Dmax of eight during the lead time. In other words we should keep a Buffer Stock = Dmax – Daverage during a lead time = 8 - 5 = 3 items.
Thus, buffer stock desired is three numbers of the given spare part.

Illustration 9:

The main shaft of an equipment has a very high reliability of 0.990. The equipment comes from Russia and has a high downtime cost associated with the failure of this shaft. This is estimated at Rs. 2 crore as the costs of sales lost and other relevant costs. However, this spare is quoted at Rs. 10 lakh at present. Should the shaft spare be procured along with the equipment and kept or not?

Answer:

The expected cost of down-time
= (Probability of failure) × (Cost when break-down occurs)
= (1 – 0.990) × (Rs. 2 crore) = Rs. 2 lakh
However, the cost of procuring the spare now is Rs. 10 lakh. Therefore, expected cost of downtime is less than the cost of spare; hence the spare need not be bought along with the equipment.

Illustration 10:

PQR company has kept records of breakdowns of its machines for 300 days work year as shown below:

No. of breakdown	Frequency in days
0	40
1	150
2	70
3	30
4	10
	300

The firm estimates that each breakdown costs Rs. 650 and is considering adopting a preventive maintenance program which would cost Rs. 200 per day and limit the number of breakdown to an average of one per day. What is the expected annual savings from preventive maintenance program?

Answer:

Step 1 : To determine the expected number of breakdowns per year:

No. of breakdowns (x)	**Frequency of breakdowns in days i.e, f(x)**	**Probability distribution of breakdowns P(x)**	**Expected value of breakdowns X P(x)**
0	40	40/300 = 0.133	Nil
1	150	150/300 = 0.500	0.500
2	70	70/300 = 0.233	0.466
3	30	30/300 = 0.100	0.300
4	10	10/300= 0.033	0.132
Total	300	1.000	1.400

Step 2 :
Total no. of breakdowns per day = 1.40
Cost of breakdown per day = 1.40 × 650 = Rs. 910
Cost of preventive maintenance programme per day = Rs. 200+ Rs. 650 = Rs. 850
Expected annual savings from the preventive maintenance programme = (910 – 850) × 300 days
= 60×300 = Rs. 18,000

Illustration 11:

A firm is using a machine whose purchase price is Rs. 15,000. The installation charges amount to Rs. 3,500 and the machine has a scrap value of only Rs. 1,500 because the firm has a monopoly of this type of work. The maintenance cost in various years is given in the following table:

Year	1	2	3	4	5	6	7	8	9
Maintenance Cost (Rs.)	260	760	1100	1600	2200	3000	4100	4900	6100

The firm wants to determine after how many years should the machine be replaced on economic considerations, assuming that the machine replacement can be done only at the year end.

Answer:

Cost of machine, C = Rs. 15,000 + Rs. 3,500 = Rs. 18,500

Scrap value, S = Rs. 1,500.

Year	**Maintenance Cost, M_1 (Rs.)**	**Cumulative Maintenance Cost, ΣM_1 (Rs.)**	**Cost of Machine – Scrap Value (Rs.)**	**Total Cost $T_{(n)}$ (Rs.)**	**Annual Cost $A_{(n)}$ (Rs.)**
(i)	**(ii)**	**(iii)**	**(iv)**	**(v)= (iii)+(iv)**	**(vi)=(v)/n**
1	260	260	17,000	17,260	17,260
2	760	1,020	17,000	18,020	9,010
3	1,100	2,120	17,000	19,120	6,373
4	1,600	3,720	17,000	20,720	5,180
5	2,200	5,920	17,000	22,920	4,584
6	3,000	8,920	17,000	25,920	4,320
7	4,100	13,020	17,000	30,020	4,288*
8	4,900	17,920	17,000	34,920	4,365
9	6,100	24,020	17,000	41,020	4,557

Lowest average cost is Rs.4,288 approx., which corresponds to n = 7 in above table. Thus machine needs to be replaced every 7th year.

Illustration 12:

A large computer installation contains 2,000 components of identical nature which are subject to failure as per probability distribution that follows:

Month End:	1	2	3	4	5
% Failure to date:	10	25	50	80	100

Components which fail have to be replaced for efficient functioning of the system. If they are replaced as and when failures occur, the cost of replacement per unit is Rs.3. Alternatively, if all components are replaced in one lot at periodical intervals and individually replace only such failures as occur between group replacement, the cost of component replaced is Rs. 1.

(a) Assess which policy of replacement would be economical.

(b) If group replacement is economical at current costs, then assess at what cost of individual replacement would group replacement be uneconomical.

(c) How high can the cost per unit in-group replacement be to make a preference for individual replacement policy?

Answer:

(a) Computation of failures & Mean life

Month (X)	**Probability of Failure (P)**	**P X**	
1	0.10	0.10	Average Life of a component = 3.35 Months
2	0.15	0.30	
3	0.25	0.75	

4	0.30	1.20	
5	0.20	1.00	
		Σpixi = 3.35 month	

Average No. of Replacements = 2000/3.35 = 597 per month

Cost of Individual Replacement = 597 × Rs. 3 = Rs. 1791 per month

Computation of expected No. of Replacements:

Month	**Expected number of components to be replaced by the month end**	
1	N1 = N0P1 = 2000 × 0.1	200
2	N2 = N0P2 + N1P1 = 2000 × 0.15 + 200 × 0.1	320
3	N3 = N0P3 + N1P2 + N2P1 = 2000 × 0.25 + 200 × 0.15 + 320 × 0.1	562
4	N4 = N0P4 + N1P3 + N2P2 + N3P1 = 2000 × 0.3 + 200 × 0.25 + 320 × 0.15 + 562 × 0.1	754.2
5	N5 = N0P5 + N1P4 + N2P3 + N3P2 + N4P1 = 2000 × 0.2 + 200 × 0.3 + 320 × 0.25 + 562 × 0.15 + 754.2 × 0.1	699.72

Computation of Average cost

Month (x)	**Cumulative number of component Replace individually by month end**	**Cost**		**Total Cost (Tc)**	**Average Cost = Tc/n**
		Individual	**Group**		
		Rs	**. Rs.**	**Rs.**	**Rs. per month**
1	200	600	2000	2600	2600
2	520	1560	2000	3560	1780
3	1082	3246	2000	5246	**1748.67***
4	1836.2	5508.6	2000	7508.6	1877.15
5	2535.92	7607.76	2000	9607.76	1921.55

Since the average cost is lowest in 3rd month, the optimal interval i.e. replacement is 3 months. Also the average cost is less than Rs. 1791 of individual replacement, **the group replacement policy is better**.

(b) Let 'K' be the cost of Individual Replacement

Month	**Average Cost of Group Replacement**	**Average cost of Individual Replacement**	**'K' Value*** (Rs.)	
1	(2000 + 200 K)/1	597 K	5.04	* To obtain the value of K use the equation Average cost of Individual Replacement = Average Cost of Group Replacement
2	(2000 + 520 K)/2	597 K	2.97	
3	(2000 + 1082 K)/3	597 K	**2.82**	
4	(2000 + 1836.2 K)/4	597 K	3.62	
5	(2000 + 2535.92 K)/5	597 K	4.45	

If group replacement is anything smaller than 2.82, then Group Replacement would be uneconomical.

(c) Let 'a' be the unit cost of Group Replacement Policy

Month	**Average Cost of Group Replacement**	**Average of Individual Replacement**	**'a' Value (Rs.)**
1	(2000 a + 600)/1	1791	0.60
2	(2000 a + 1560)/2	1791	1.01
3	(2000 a + 3246)/3	1791	**1.06**
4	(2000 a + 5508.6)/4	1791	0.83
5	(2000 a + 7607.76)/5	1791	0.67

When unit cost is more than Rs. 1.06 then Individual Replacement policy would be better.

Illustration 13:

An electric company which generates and distributes electricity conducted a study on the life of poles. The repatriate life data are given in the following table:

Life data of electric poles

Year after installation:	1	2	3	4	5	6	7	8	9	10
Percentage poles failing:	1	2	3	5	7	12	20	30	16	4

- If the company now installs 5,000 poles and follows a policy of replacing poles only when they fail, how many poles are expected to be replaced each year during the next ten years?
  To simplify the computation assume that failures occur and replacements are made only at the end of a year.
- If the cost of replacing individually is Rs. 160 per pole and if we have a common group replacement policy it costs Rs. 80 per pole, find out the optimal period for group replacement.

Answer:

Chart showing Optimal Replacement Period

Average life of the pole - 1 × 0.01 + 2 × 0.02 + 3 × 0.03 + 4 × 0.05 + 5 × 0.07 + 6 × 0.12 + 7 × 0.20 + 8 × 0.3 + 9 × 0.16 + 10 × 0.04 = 7.05 years.

No. of poles to be replaced every year = 5000 / 7.05 = 709

Average yearly cost on individual replacement = 709 × Rs.160 = Rs.1,13,440.

Group Replacement: Initial Cost = 5,000 × Rs.80 = Rs.4,00,000.

Year	**No. of poles to be replaced**	**Yearly cost of individual replacement @ Rs. 160/pole (Rs.)**	**Cumulative cost of individual replacement (Rs.)**	**Total cost of individual replacement as well as group replacement (Rs.)**	**Average Annual Cost = Total Cost / Year (Rs.)**
1	5,000 × 0.01 = 50	8,000	8,000	4,08,000	4,08,000
2	5,000 × 0.02 + 50 × .01 = 101	16,160	24,160	4,24,160	2,12,080
3	5,000 × 0.03 + 50 × 0.02 + 101 × 0.01 = 152	24,320	48,480	4,48,480	1,49,493
4	5,000 × 0.05 + 50 × 0.03 + 101 × 0.02 + 152 × 0.01 = 256	40,960	89,440	4,89,440	1,22,360
5	5,000 × 0.07 + 50 × 0.05 + 101 × 0.03 + 152 × 0.02 + 256 × 0.01 = 362	57,920	1,47,360	5,47,360	1,09,472
6	5,000 × 1.2 + 50 × 0.07 + 101 × 0.05 + 152 × 0.03 + 256 × 0.02 + 362 × 0.01 = 6023	9,63,680	11,11,040	15,11,040	2,51,840

Optimal replacement at the end of the 5th year.

Illustration 14:

Product A has a Mean Time Between Failures (MTBF) of 30 hours and has a Mean Time To Repairs (MTTR) of 5 hours. Product B has a MTBF of 40 hours and has a MTTR of 2 hours.

(i) Which product has the higher reliability?

(ii) Which product has greater maintainability?

(iii) Which product has greater availability?

Answer:

(i) Product B, with higher MTBF (i.e. 40 hours) than Product A (i.e. 30 hours), is more reliable since it has lesser chance of failure during servicing.

(ii) By MTTR we mean the time taken to repair a machine and put it into operation. Thus Product B, with lesser MTTR (i.e., 2 hours) than Product A (i.e., 5 hours), has greater maintainability.

(iii) Availability of a machine/product = $\frac{\text{MTBF}}{\text{MTBF + MTTR}}$

Therefore, Availability of Product A = 30/ (30+5) = 30/35 = 85.714% Availability of Product B = 40/(40 + 2) = 40/42 = 95.238%

Hence, Product B has more availability.

Illustration 15:

Maharashtra Trucking Company (MTC) has a fleet of 50 trucks. The past data on the breakdown of the trucks show the following probability distribution (for a new truck as well as for one which has been repaired after a breakdown).

Months after Maintenance	Probability of Breakdown
1	0.10
2	0.20
3	0.30
4	0.40

Each breakdown costs Rs. 3,000 on an average; which includes cost of time lost and cost of materials and manpower.

The manager of MTC knows the importance of preventive maintenance. He estimates the costs of the preventive maintenance to be Rs. 500 per such preventive action. What should be the appropriate maintenance policy in terms of the mix of preventive and breakdown maintenance

Answer:

First, let us compute the cost of a totally breakdown maintenance policy.

The expected number of months between failures

= 0.1 (1) + 0.2 (2) + 0.3 (3) + 0.4 (4) = 3.0

Cost per month of totally breakdown maintenance policy=

$$\frac{\text{(No. of trucks) (Cost per breakdown)}}{\text{(Expected number of months between failure)}}$$

$$= \frac{(50)(\text{Rs. } 3000)}{(3.0)} = \text{Rs. } 50{,}000$$

Now let us compute the costs of different periodicities of preventive maintenance.

(i) Preventive maintenance (PM) period one month

No. of breakdowns within the period of one month:

B1 = (50) × (0.1) = 5

Cost of breakdown = 5 × Rs. 3000 = Rs. 15,000

Cost of preventive maintenance = Rs. 500 × 50 = Rs. 25,000

Total Cost during the PM period = Rs. 40,000

Therefore, cost per month for this policy is

= 40,000 ÷ 1 = Rs. 40,000

(ii) Preventive maintenance (PM) period two months

No. of breakdowns within 2 months:

B2 = (50) × (0.1 + 0.2) + (50) × (0.1) × (0.1) = 15.5

Cost of breakdown = (15.5) × Rs. 3000 = Rs. 46,500

Cost of prev. maintenance = Rs. 500 × 50 = Rs. 25,000

Total cost during the PM period = Rs. 71,500

Therefore, cost per month for this policy:

Rs. 71,500 ÷ 2 months = Rs. 35,750

(iii) Preventive maintenance period 3 months

No. of breakdowns within 3 months:

B3 = (50)× (0.1 + 0.2 + 0.3) + (50 × 0.1) (0.1 + 0.2) + (50 × 0.1 × 0.1) (0.1)

= 30 + 1.5 + 0.05 = 31.55

Cost of breakdown = 31.55 × Rs. 3000 = Rs. 94,650

Cost of preventive maintenance = 50 × Rs. 500 = Rs. 25,000

Total = Rs. 1,19,650

Therefore, cost per month for this policy

= Rs. 1,19,650 ÷ 3 months = Rs. 39,883.33

(iv) Preventive maintenance period 4 months

No. of breakdowns within 4 months

B4 = [(50) × (1.0)] + [(50) × (0.1) × (0.1 + 0.2 + 0.3) + (50× 0.1 × 0.1) × (0.1 + 0.2) + (50 × 0.1 × 0.1 × 0.1) × (0.1) + (50 × 0.1 × 0.2) × (0.1)]+ [(50 × 0.2) × (0.1 + 0.2) + (50 × 0.2 × 0.1) × (0.1)] + [(50 × 0.3 × (0.1)] = 57.855

Cost of breakdown = (57.855) × (Rs. 3,000) = Rs. 1,73,565

Cost of preventive maintenance = 50 × Rs. 500 = Rs. 25,000

Total = Rs. 1,98,565

Therefore, cost per month for this policy is Rs. 1,98,565 ÷ 4 months = Rs. 49,641.25

Comparing the costs per month of different policies, we see that the policy of preventive maintenance every two months is the most economic policy.